

# ON $\LaTeX$ AND THE ROLE OF THE INFINITE IN MATHEMATICAL REASONING

DAVID HILBERT AND STEPHEN G. SIMPSON

*Dedicated to Professor Donald Knuth on the occasion of his 100th birthday*

ABSTRACT. This is a sample document illustrating how to use  $\LaTeX$  in the Penn State Math Department. We also attempt to clarify the mysteries of the infinite.

## 1. INTRODUCTION

This document was prepared with  $\LaTeX$  using the AMS article style. By comparing this printed document with the raw document file from which it was generated, you can learn how to use  $\LaTeX$  to typeset mathematical documents.

To start a new paragraph, skip a line.

Many people regard the infinite as a great mystery. In this paper we shall try to end the mystery once and for all, so that mathematicians will never again have to bother about it.

The set of real numbers is denoted  $\mathbb{R}$ . One of its subsets is  $\emptyset$ , *i.e.* the empty set.

We end this introductory section with a brief outline of the rest of the paper. Section 2 reviews a great number of important concepts and results. Then Section 3 presents our new contributions. At the end of the paper comes a bibliography, also known as a list of references.

## 2. PRELIMINARIES: THE MANY FACES OF $\infty$

The purpose of this section is to review some well-known concepts and results. Most of these results are taken from the local newspaper [2]. The O. J. Simpson trial is very important. Cousin OJ seems to be in a lot of trouble. See also the front page of [2].

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The infinite has many guises. One popular notation for the infinite is  $\infty$ . Cantor's aleph notation provides an important refinement:  $\aleph_0$  denotes countable infinity, while other infinities are denoted  $\aleph_\alpha$  where  $\alpha$  is an ordinal number greater than 0.

### 3. THE MAIN MATHEMATICAL RESULTS

We begin with the following lemma.

**Lemma 3.1.** *For all ordinal numbers  $\alpha$ , we have  $\aleph_\alpha < 2^{\aleph_\alpha}$ .*

*Proof.* The proof is obvious by diagonalization. □

An immediate consequence of the previous lemma is:

**Theorem 3.2.** *We have  $\aleph_0 < 2^{\aleph_0}$ .*

**Corollary 3.3** (Cantor's Theorem). *The real number system,  $\mathbb{R}$ , is uncountable.*

*Proof of Cantor's Theorem.* Immediate from Theorem 3.2. □

This completes our discussion of Cantor's Theorem.

An interesting equation is

$$(1) \quad (A + B)^n = \sum_{i=0}^n \binom{n}{i} A^i B^{n-i}.$$

Note that equation (1) is sometimes called the Binomial Theorem.

### 4. SOME FEATURES OF $\LaTeX$

$\LaTeX$  has a lot of author-friendly features. Among them are:

1. a well-written reference manual;
2. automatic numbering and cross referencing of
  - (a) sections
  - (b) equations
  - (c) tables
  - (d) figures
  - (e) *etc.*
3. convenient bibliographic citations;
4. special environments for tables and figures.

For instance, Table 1 is a simple table, produced using the `tabular` environment.

TABLE 1. Grades earned by political leaders

Last Name	Score	Letter Grade
Clinton	95	A
Dole	95	A
Gingrich	95	A

## 5. CONCLUSIONS

We hope that this article has illustrated how easy it is to use  $\text{\LaTeX}$ . It is also clear from Theorem 3.2 that the infinite is a subject that may or may not be worthy of the attention of all mathematicians.

## REFERENCES

- [1] D. König, *Theorie der Endlichen und Unendlichen Graphen*, Akademische Verlagsgesellschaft, Leipzig, 1936, reprinted by Chelsea Publishing Company, New York, 1950, 258 pages.
- [2] The Centre Daily Times.

UNIVERSITY OF GÖTTINGEN, GÖTTINGEN, GERMANY  
*E-mail address:* `dh@math.gottingen.edu.de`

UNIVERSITY OF GÖTTINGEN, GÖTTINGEN, GERMANY  
*Current address:* Department of Mathematics, Pennsylvania State University,  
University Park, State College PA 16802  
*E-mail address:* `simpson@math.psu.edu`