Development of Rabin's Choice Coordination Algorithm in Event-B

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AVoCS'10, 21st-23rd September, 2010
Düsseldorf, Germany

(part of the work is supported by DEPLOY, an FP7 European Project)

Certain v.s. Almost-Certain Termination

- Consider tossing a fair coin c until it comes up head (H).

while $c = T$ do $c :\in \{H, T\}$ end	while $c = T$ do $c := H \oplus_{1/2} T$ end
Demonic non-termination	Probabilistic termination

- Technique: loop variant on some well-founded order.
- Certain termination: Every iteration must decrease the loop variant.
- Almost-certain termination ([MM05])[1]:
  - Every iteration might decrease the loop variant.
  - The variant is bounded above.
  - The probability needs to be proper (bounded away from 0 and 1).

[1] [MM05] C. Morgan, A. McIver. *Abstraction, Refinement and Proof for Probabilistic Systems*. 2005.

Qualitative Reasoning in Event-B

- Introduced in [HH07][2] .
- Introduction of probabilistic events.
- Behave (almost) the same as standard non-deterministic events, e.g. invariant preservation proof obligations.
- Behave differently for convergence proof obligations.

[2] [HH07] S. Hallerstede, T. Hoang. Qualitative Probabilistic Modelling in Event-B. In *iFM 2007*

Our Contribution

Questions

- Probabilistic events and Event-B's developments with refinement?
- How to construct an probabilistic lexicographic variant?

Contribution

- An approach for developing almost-certain termination systems.
  - Extended Rodin Platform for tool support.
  - Formalised Rabin's Choice Coordination algorithm.

Background. Event-B

- A modelling notation for discrete transition systems.
- Models (machines) contain variables, invariants and events
- Events contain parameters, guards and actions

E
  status *ordinary / convergent / anticipated*
  any t **where**
    $G(t, v)$
  then
    $v :| S(t, v, v')$
  end

Convergence in Event-B

- A variant $V(v)$ is proposed.
- The variant must be a finite set or a natural number.
- Every convergent event must decrease the variant.
- Every anticipated event must not increase the variant.
- Combination with refinement: lexicographic variant.
  - Model M_0: E_1 is convergent and E_2 is anticipated with variant V_1.
  - Model M_1 refines M_0: E_2 is convergent with variant V_2.
  - (V_1, V_2) is a lexicographic variant with V_1 has higher precedence.

$$(V_1, V_2) < (V_1', V_2') \quad \Leftrightarrow \quad (V_1 < V_1') \vee (V_1 = V_1' \wedge V_2 < V_2')$$

Probabilistic Events in Event-B

```
E
   status   probabilistic
   any   t   where
      G(t, v)
   then
      v :| S(t, v, v')
   end
```

- The variant $V(v)$ is bounded above by a constant B.
- The event might decrease the variant $V(v)$.

Probabilistic Lexicographic Variant

Constructing lexicographic variant, e.g. (V_1, V_2):

- Requires refinement.
  - Standard refinement does not preserve almost-certain termination.

```
ae
  status  probabilistic
  any  ...  where
    ...
  then
    v :∈ {good, bad}
  end
```

```
ce
  refines  ae
  status  probabilistic
  any  ...  where
    ...
  then
    v := bad
  end
```

  - To restrict refinement.
- (V_1, V_2) needs to be bounded above.
  - All sub-variants need to be bounded above.
    (including the variant for proving standard convergence)

Our Approach

Goal

To prove that condition P holds eventually with probability 1 at the end of a program.

The Approach

1. Establish the model of the program contains:
   - an observer event[a]

     obs $\widehat{=}$ **when** P **then** skip **end**

   - several anticipated events $E_1, \ldots, E_n$.
2. Prove that eventually only obs is enabled:
   - $E_1, \ldots E_n$ are convergent (either probabilistic or standard).
   - The system is deadlock-free.

[a] [HKBA09] T.S. Hoang, H. Kuruma, D. Basin and J-R. Abrial. Developing Topology Discovery in Event-B. 2009

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Choice Coordination Problem and Rabin's Algorithm

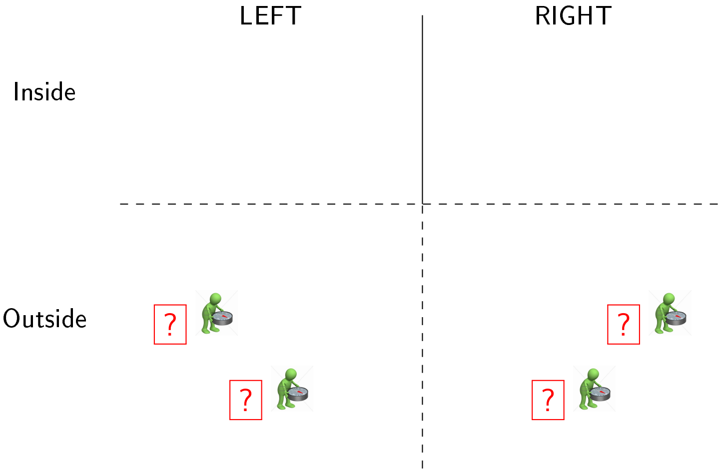
Choice Coordination Problem

- Given n processes $P_1, \ldots, P_n$.
- Given k alternatives $A_1, \ldots, A_k$.
- Aim: Processes reach a common choice out of the alternatives.
- Constraints: Processes must not communicate directly.

Rabin's Algorithm

- The protocol uses k shared variables, one for each alternative.
- A process assume to access and modify a shared variable atomically.
- A simplified version of the algorithm by McIver/Morgan with $k = 2$.

Algorithm Context



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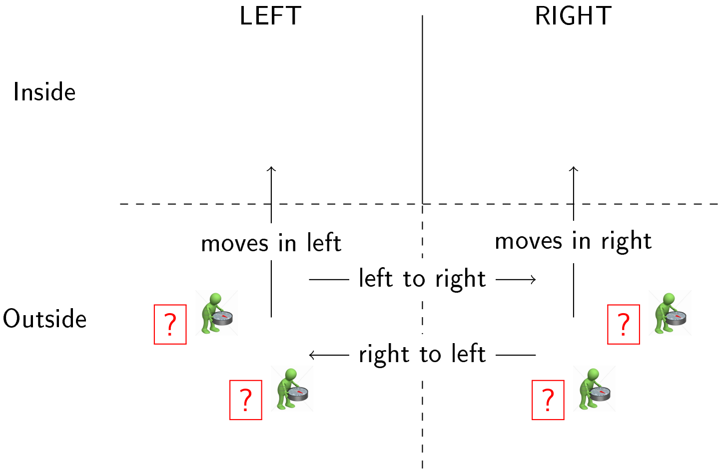
Algorithm Context



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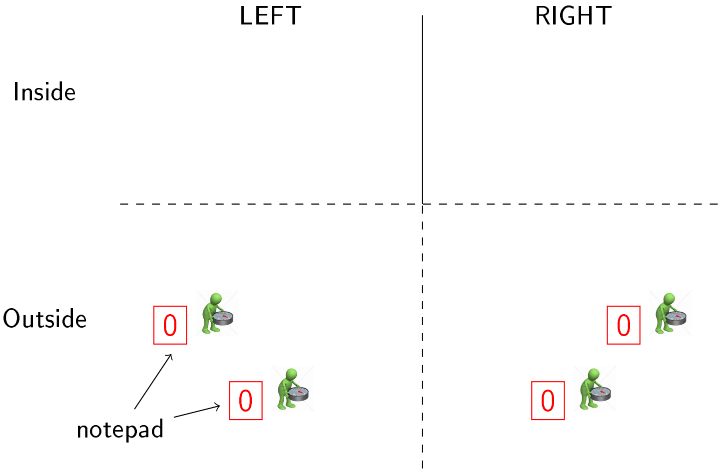

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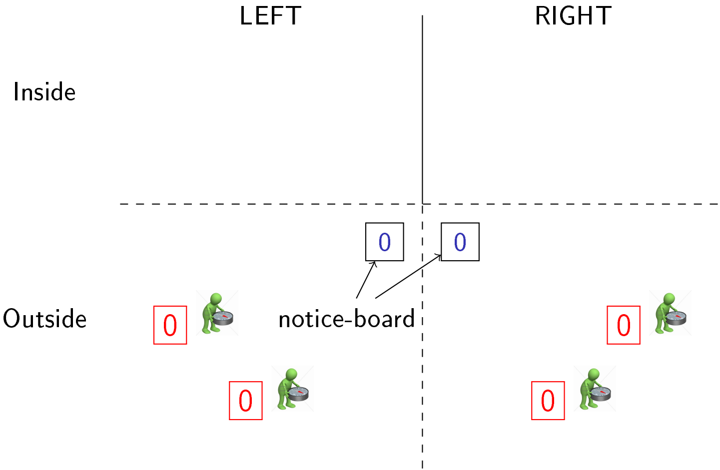


Formal Model. The State

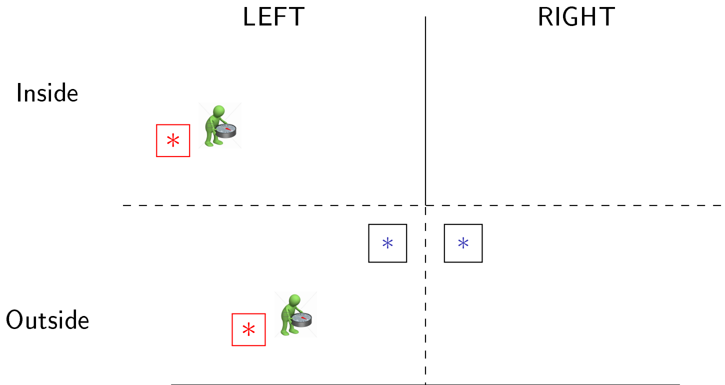
variables: $lin, rin,$
$lout, rout,$
L, R, np

invariants:

inv0_3	:	$lin = \varnothing \lor rin = \varnothing$
inv1_1	:	$partition(T, lin, rin, lout, rout)$
inv2_1	:	$L \in \mathbb{N}$
inv2_2	:	$R \in \mathbb{N}$
inv2_3	:	$np \in T \to \mathbb{N}$

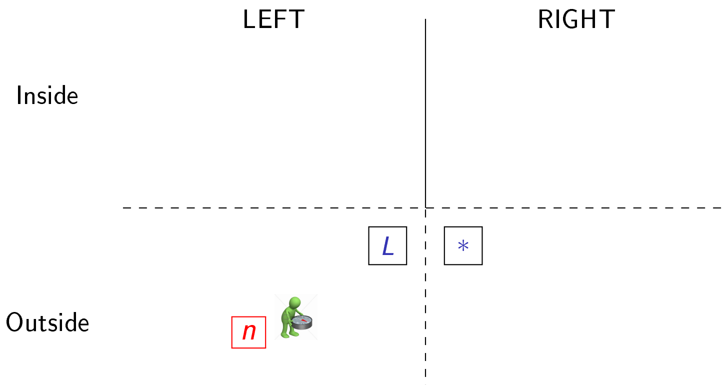
init
begin
$lin := \varnothing$
$rin := \varnothing$
$lout, rout :\mid lout' = T \setminus rout'$
$L := 0$
$R := 0$
$np := T \times \{0\}$
end

Algorithm. A Tourist Moves In (First Case)



If there are some tourists inside,
he goes in

Algorithm. A Tourist Moves In (Second Case)



If there is no one inside and $L < n$,
he goes in

Algorithm. A Tourist Alternates (First Case)

LEFT | RIGHT

Inside

L | $*$

Outside

n

If there is no one inside and $L > n$,
he replaces n by L on his notepad

Algorithm. A Tourist Alternates (Second Case)

LEFT | RIGHT

Inside

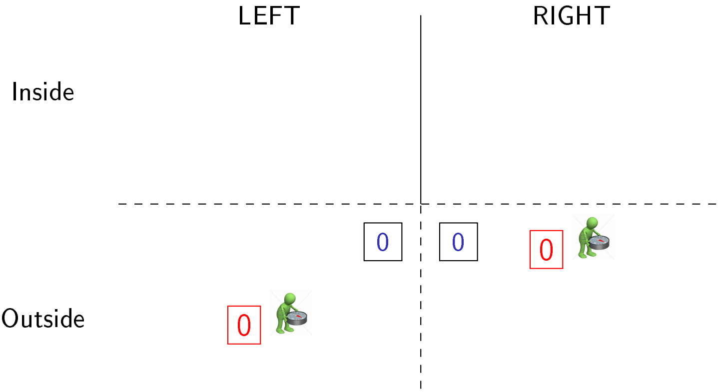
L | $*$

Outside

n

If there is no one inside and $L = n$,
he first tosses a coin and choose a number L'

Animation with Two Tourists



There are two tourists.
Assume the tourist on the LEFT has the turn.

Algorithm Intuition

...	...
Round 3	7
	6
Round 2	5
	4
Round 1	3
	2
Round 0	1
	0

- Conjugate of an even number n is $n+1$.
- Conjugate of an odd number n is $n-1$.
- The algorithm contains several rounds.
- In each round, each notice board is chosen probabilistically in the next pair.
- The algorithm terminates when the values of the notice boards are different in the same round.

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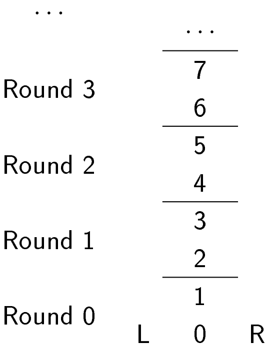

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Refinement Strategy

- Initial model: introduce the set of tourists inside: *lin* and *rin*.
- 1st Ref.: introduce the set of tourists outside: *lout* and *prout*.
- 2nd Ref.: introduce Rabin's algorithm including the noticeboards (*L*, *R*) and tourists' notepads (*np*).
- 3rd–6th Refs.: prove convergence property.
  - A lexicographic variant with 2 layers [MM05].
  - We used both finite set and natural number variants.
  - Split and merge of events: Simpler proofs..
- 7th Ref.: prove deadlock-freeness.

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Proof Statistics

Model	Total	Auto.(%)	Man.(%)
Initial model	6	6(100%)	0(N/A)
1st Refinement	8	7(88%)	1(12%)
2nd Refinement	63	49(78%)	14(23%)
Outer variant	54	29(54%)	25(46%)
Inner variant	11	8(73%)	3(27%)
Deadlock freedom	4	0(0%)	4(100%)
Total	146	99(68%)	47(32%)

Conclusion

- An approach for developing almost-certain termination programs.
  - probabilistic lexicographic variant.
  - Practical tool support.

Future work

- Improve tool support.
- Verify other examples, e.g. IEEE1394 protocol.
- Elaborate refinement while preserving probabilistic convergence.

For Further Reading I

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