Qualitative Reasoning for the Dining Philosophers

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Outline

Motivation

- Probabilistic solution for the dining philosophers.
- Proof from McIver and Morgan: Fairness + probability
- Here: probability only.
- Requirements:
  - simplicity
  - must yield a method
- Approach:
  - create a proof
  - not yet worry too much about the semantic models.
  - do that when we are sure the proof is good enough.

The Dining Philosophers

- A number of philosophers sit at a round table.
- Between each adjacent pair of philosopher is a single fork.
- In order to eat, each philosopher need two forks on both sides.
- When hungry, a philosopher might want to pick up a fork, but this might already be taken by his neighbouring philosopher.
- There is a possibility of deadlock or livelock.
- There are deterministic solutions, e.g. using a waiter to break symmetry.
- We consider a symmetric probabilistic solution.

A Probabilistic Algorithm

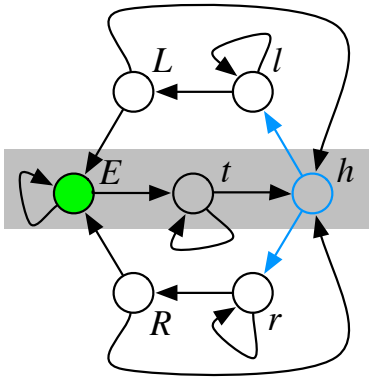


Figure: Actions of a philosophers

Fairness Assumption

Fairness assumption

Every philosopher is scheduled infinitely often with probability one.

Overall system

Some philosophers are hungry;
while "No philosopher is eating" **do**
  Schedule one of the philosopher fairly
end

Standard Event Convergent in Event-B

Intuitively

Event must decrease the variant.

More precisely

evt
  any x **where**
    $G(x, v)$
  then
    $v :| S(x, v, v')$
  end

variant: $V(v)$

$$\begin{array}{l} \ldots \\ G(x, v) \\ \vdash \\ \forall v' \cdot S(y, v, v') \Rightarrow V(v') \subset V(v) \end{array}$$

Probabilistic Event Convergent in Event-B

Intuitively

Event might decrease the variant.

More precisely

evt
  any x **where**
    $G(x, v)$
  then
    $v \oplus| S(x, v, v')$
  end

variant: $V(v)$

$$\begin{array}{l} \ldots \\ G(x, v) \\ \vdash \\ \exists v' \cdot S(y, v, v') \wedge V(v') \subset V(v) \end{array}$$

The State

Overall system

Some philosophers are hungry;
while "No philosopher is eating" **do**
  Schedule one of the philosopher probabilistically
end

Variables and invariants

variables: h, t, e

invariants:
$partition(P, h, t, e)$

init
  begin
    $h, t :| partition(P, h', t') \wedge h' \neq \varnothing$
    $e := \varnothing$
  end

The Events

The events

eats
  any p **where**
    $p \in h$
  then
    $e := e \cup \{p\}$
    $h := h \setminus \{p\}$
  end

thinks
  any p **where**
    $p \in e$
  then
    $t := t \cup \{p\}$
    $e := e \setminus \{p\}$
  end

getsHungry
  any p **where**
    $p \in t$
  then
    $h := h \cup \{p\}$
    $t := t \setminus \{p\}$
  end

Refinement strategy (1)

Strategy

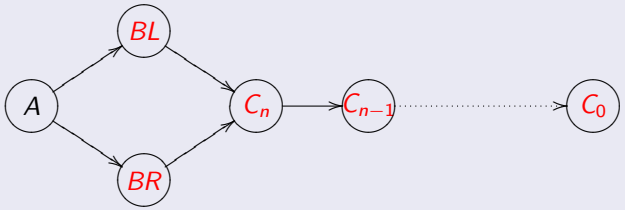
- Gradually introduce the algorithm: new variables/events are added.
- Prove that events other than eats are (probabilistic) convergent.
- System is deadlock-free.

Consequence

Eventually some (hungry) philosopher will eat.

Refinement strategy (2)

The Lexicographic Variant



Probabilistic Convergent in BR and BL Phase

choose event: Pick-up left or right fork first

choose
  any p **where**
    $p \notin l$
    $p \notin r$
    ...
  then
    $l, r \oplus\mid (l' = l \cup \{p\} \wedge r' = r) \vee$
      $(r' = r \cup \{p\} \wedge l' = l)$
  end

variant: $P \setminus r$

$p \notin l$
$p \notin r$
...
$\vdash$
$\exists l', r' \cdot$
  $((l' = l \cup \{p\} \wedge r' = r) \vee$
   $(r' = r \cup \{p\} \wedge l' = l)) \wedge$
  $P \setminus r' \subset P \setminus r$

Probabilistic Convergent in C_n Phases

Pick up a (left) fork

chooseLeft
  any p **where**
    $p \in l$
    ...
  then
    ...
  end

dropLeft
  any p **where**
    $p \in L$
    ...
  then
    ...
  end

...

Difficulties

- The probabilistic choice is associated with the parameter p.
- The reasoning must taken into account all the actions that a particular philosopher can do.
- Need to prove: There exists a philosopher such that he can always act, and any action that he made decreases the variant.

A Possible Solution

evt_i
  any t **where**
    $G_i(t, v)$
  then
    $v :\mid Q_i(t, v, v')$
  end

variant: V

witness: $W(t, v)$

- Sketch of probabilistic termination witness for t, say $W(t, v)$.
- Sketch of the proof obligations.
  1. Existent of witness: $I(v) \Rightarrow (\exists t \cdot W(t, v))$.
  2. Given the witness, at least one probabilistic event is enable. $I(v) \wedge W(t, v) \Rightarrow G_1(t, v) \vee \ldots \vee G_n(t, v)$
  3. For any probabilistic event evt_i, it decreases the variant V: $I(v) \wedge W(t, v) \wedge G_i(t, v) \wedge Q_i(t, v, v') \Rightarrow V(v') \subset V(v)$

What About Refinement

- Refinement can reduce non-determinism.
- Qualitative termination is not preserved through this type of refinement.
- We need to have additional proof obligation(s) for preserving qualitative termination.
- But this should be simple and usable.

For Further Reading I


S. Hallerstede and T.S. Hoang
Qualitative Probabilistic Modelling in Event-B,.
IFM 2007.


A. McIver and C. Morgan.
Abstraction, Refinement and Proof for Probabilistic Systems,
Chapter 3 — Case studies on probabilistic termination.
2005.