Event-B Decomposition for Parallel Programs

Thai Son Hoang

Department of Computer Science
Swiss Federal Institute of Technology Zürich (ETH Zürich)

Dagstuhl Seminar, 13th-18th September, 2009

Outline

Motivation

- Parallel programs.
- Event-B for discrete transition systems.
- Work on "interference-free" (by S. Owicki and D. Gries).
- Work on Rely/Guarantee (by C. Jones)
- Example: FindP program.

Overview

	1	2	3	$\cdots$	M
ARRAY	$\neg$P	$\neg$P	P	$\cdots$	P

FindP Program

Finding the first index k of an array $ARRAY$, if there is one, such that $ARRAY(k)$ satisfied some property P. Otherwise, return $M + 1$.

- The program use two parallel processes
  to check two parts $PART1$ and $PART2$ of the array separately.
- Each process publishes the first index that it finds.

FindP with Parallel Processes

Main programs

$index1, index2 := min(PART1), min(PART2);$
$publish1, publish2 := M + 1, M + 1;$
process1 || **process2**;
$k := min(\{publish1, publish2\})$

Process: process1

while $index1 < min(\{publish1, publish2\})$ **do**
  if $ARRAY(index1) = \mathrm{TRUE}$ **then**
    $publish1 := index1$
  else
    $index1 :=$ `the-next-index-in-PART1`
  end
end

Unfolding process1

Process: process1

$1 : (read)$ $read1 := publish2;$
$2 :$ **if** $index1 < min(\{publish1, read1\})$ **then**
  if $ARRAY(index1) = \mathrm{TRUE}$ **then**
$(found)$ $publish1 := index1 \;||\; goto\ 3(end);$
  else
(inc) $index1 :=$ `next-in-PART1` $||\; goto\ 1(read);$
  end
else
(not_found) $goto\ 3(end)$
end
$3 : (end)$

Ideas for Decomposition

- Specify the program globally.
- Decomposing the program into different processes: main, process1, process2.

The Context

The Context

1	2	3	···	M
F	F	T	···	F

constants: $M, ARRAY$

axioms:
axm0_1: $M \in \mathbb{N}_1$
axm0_2: $ARRAY \in 1\,..\,M \rightarrow \mathrm{BOOL}$

The Specification

The state and events

1	2	3	⋯	M
F	F	T	⋯	F

variables: *result*

invariants:
inv0_1: $result \in \mathbb{Z}$

init
begin
$result :\in \mathbb{Z}$
end

final
any k **where**
$k \in 1 \,..\, M+1$
$\forall j \cdot j \in 1 \,..\, k-1 \Rightarrow ARRAY(j) = \mathrm{FALSE}$
$k \neq M+1 \Rightarrow ARRAY(k) = \mathrm{TRUE}$
then
$result := k$
end

The Refinement

The published values of two processes

variables: $\ldots, finish1, finish2, publish1, publish2$

init
begin
…
$finish1 := FALSE$
$finish2 := FALSE$
$publish1 := M+1$
$publish2 := M+1$
end

The Final Event

(abs_)final
any k **where**
$k \in 1 \,..\, M+1$
$\forall j \cdot j \in 1 \,..\, k-1 \Rightarrow ARRAY(j) = \mathrm{FALSE}$
$k \neq M+1 \Rightarrow ARRAY(k) = \mathrm{TRUE}$
then
$result := k$
end

(conc_)final
refines (*abs_*)*final*
when
$finish1 = \mathrm{TRUE}$
$finish2 = \mathrm{TRUE}$
with
$k = min(\{publish1, publish2\}$
then
$result := min(\{publish1, publish2\})$
end

The Invariants

The invariants

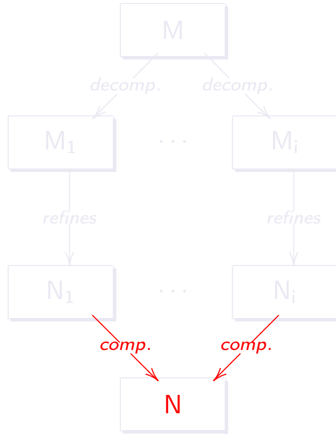
invariants:
$publish1 \neq M+1 \Rightarrow finish1 = \mathrm{TRUE}$
$publish1 \neq M+1 \Rightarrow publish1 \in PART1$
$publish1 \neq M+1 \Rightarrow ARRAY(publish1) = \mathrm{TRUE}$
$publish1 \neq M+1 \Rightarrow (\forall i \cdot i \in PART1 \land i < publish1 \Rightarrow ARRAY(i) = \mathrm{FALSE})$
$finish1 = \mathrm{TRUE} \land publish1 = M+1 \Rightarrow$
$(\forall i \cdot i \in PART1 \land i < publish2 \Rightarrow ARRAY(i) = \mathrm{FALSE})$
…

The Abstract Events for **process1**.

found_1
any k **where**
$finish1 = \mathrm{FALSE}$
$k \in PART1$
$ARRAY(k) = \mathrm{TRUE}$
$\forall i \cdot i \in PART1 \land i < k \Rightarrow ARRAY(i) = \mathrm{FALSE}$
$publish1 = M + 1$
then
$finish1 := \mathrm{TRUE}$
$publish1 := k$
end

not_found_1
when
$finish1 = \mathrm{FALSE}$
$\forall i \cdot i \in PART1 \land i < publish2 \Rightarrow ARRAY(i) = \mathrm{FALSE}$
then
$finish1 := \mathrm{TRUE}$
end

Overview



Shared Variables Decomposition in Event-B

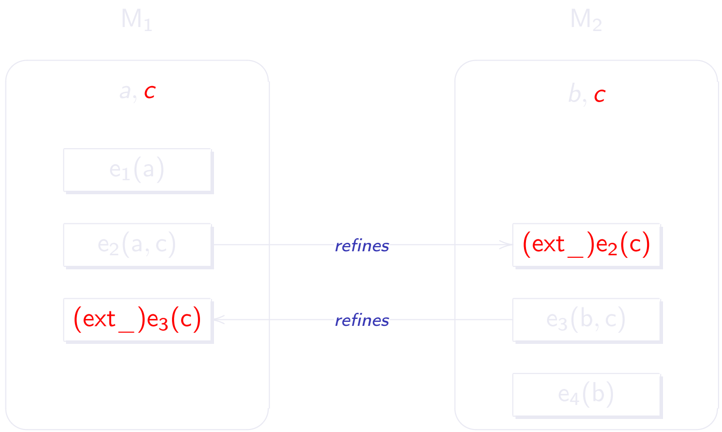
(Also called A-Style decomposition)

- Sub-models shared variables.
- The set of internal events of sub-models are disjoint.
- Each models having a set of external events
  to model the effect of these events on shared variables.

An Example (1)

- Assume model M has the following events:
  $e_1(a)$, $e_2(a, c)$, $e_3(b, c)$, $e_4(b)$.
- Events partition:
  - M_1: e_1, e_2.
  - M_2: e_3, e_4.
- Variables distribution (calculated from events partition):
  - M_1: Private variable a, shared variable c.
  - M_2: Private variable b, shared variable c.
- Result:
  - M_1: Internal events $e_1(a)$, $e_2(a, c)$, external event (ext_)$e_3(c)$.
  - M_2: Internal events $e_3(b, c)$, $e_4(c)$, external event (ext_)$e_2(c)$.

An Example (2)



Constructing External Events

Informally ...

(ext_)e_2 is the projection of e_2
on the state containing only external variables c.

More precisely ...

$M_1(a, c)$	$M_2(b, c)$
e_2 **any** t **where** $G(t, a, c)$ **then** $a, c :\mid Q(t, a, c, a', c')$ **end**	(ext_)e_2 **any** t, a **where** $G(t, a, c)$ **then** $c :\mid \exists a' \cdot Q(t, a, c, a', c')$ **end**

Back to FindP

Decomposition Ideas

main: final

process1: not_found_1 and found_1.

process2: not_found_2 and found_2.

Refinement Strategy for **process1**

Constraints during refinement

- Shared variables and external events cannot be refined.
- External events must preserve the newly introduced invariants.

1. 1st Ref.: Introducing the local index of the array.
2. 2nd Ref.: Introducing the read value.
3. 3rd Ref.: Introducing the address counter for sequencing the events.

Related Work (1)

- Notion "Interference-free" from Owicki-Gries.

  Consider a proof of $\{P\}S\{Q\}$ and a statement T with precondition $pre(T)$, T does not interfere with $\{P\}S\{Q\}$ if

  Inf1 $\{Q \wedge pre(T)\}T\{Q\}$.

  Inf2 Let S' be any statement within S, then $\{pre(S') \wedge pre(T)\}T\{pre(S')\}$

- Comparing the work:
  - S is an internal event of **process1**.
  - T is an external event of **process1**.
  - The condition **Inf1** is proved at the level before decomposing.
  - S' is introduced during the refinement of S.
  - $pre(S')$ are the invariants introduced during refinement.
  - The condition **Inf2** is proved during refinement: external event preserves invariants.

Related Work (2)

- Rely/Guarantee method from Jones.
  - Extending the Hoare's triple to include the Rely/Guarantee conditions R and G, i.e. $\{P, R\}S\{G, Q\}$.
  - An example rule for parallel composition

$$\textbf{PAR-I}\quad \frac{\begin{array}{l} R \vee G_1 \Rightarrow R_2 \\ R \vee G_2 \Rightarrow R_1 \\ G_1 \vee G_2 \Rightarrow G \\ \{P, R_1\}S_1\{G_1, Q_1\} \\ \{P, R_2\}S_2\{G_2, Q_2\} \end{array}}{\{P, R\}S_1 || S_2\{G, Q_1 \wedge Q_2\}}$$

Related Work (3)

- The rely/guarantee condition are relations over the two states.
- A pair of external/internal events
  - External event: Rely condition.
  - Internal event: Guarantee condition.
- $\Rightarrow$ relation of rely/guarantee conditions becomes event refinement.
- The generated pair of external/internal events satisfies the rules for parallel composition.
- However, this generated external events might be too "concrete".
- In the FindP example, the external events just need to guarantee to decrease the published value monotonically.
- User-defined external events.

For Further Reading I

C. Jones.
Splitting atoms safely,.
Theor. Comput. Sci. 2007.

S. Owicki and D.Gries.
An Axiomatic Proof Technique for Parallel Programs I.
Acta Inf. 6, 1976.