A Step-wise Development Method with Progress Concerns

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Formal Systems Development using Refinement

- To develop a system **M** satisfying property ϕ, i.e., $\mathbf{M} \models \phi$.
  - **M**: some transition system
  - ϕ: some logical formula
- The main challenge: the complexity of the system.
- Refinement allows the step-by-step design of the system.

Refinement

The UNITY way vs. the Event-B way

- UNITY: Refines the formulae.

$$\overbrace{\phi \Leftarrow \phi_1 \Leftarrow \ldots \Leftarrow \phi_n}^{\text{Refinement}} \; \underbrace{\dashv \; \mathbf{M}}_{\text{Translation}}$$

  - Cons: Hard to understand the choice of refinement.
- Event-B: Refines transition systems.

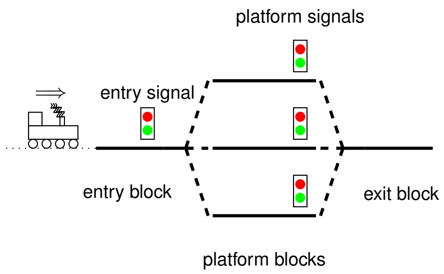
$$\underbrace{\phi \dashv \mathbf{M}_0}_{\text{Verification}} \sqsubseteq \overbrace{\mathbf{M}_1 \ldots \sqsubseteq \mathbf{M}}^{\text{Refinement}}$$

  - Cons: No support for liveness properties.

The Unit-B Modelling Method

- Inspired by UNITY and Event-B.
- Support the reasoning of liveness properties (UNITY).
- Refinement of transition systems (Event-B style).
- Developments using Unit-B are
  guided by both safety and liveness requirements.

Running Example. A Signal Control System



SAF 1 There is at most one train on each block

LIVE 2 Each train in the network eventually leaves

Unit-B Models – Discrete Transition Systems

- States are captured by variables v.
- Transitions are modelled by guarded and scheduled events.

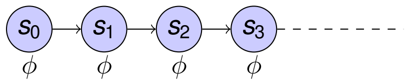
Traces and the Language of Temporal Logic

A trace σ is a (finite or infinite sequence of states)

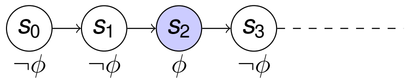
$$\sigma = s_0, s_1, s_2, s_3, \ldots$$

- A (basic) state formula P is any first-order logic formula,
- The basic formulas can be extended by combining the Boolean operators $(\neg, \wedge, \vee, \Rightarrow)$ with temporal operators:

always: $\Box\,\phi$



eventually: $\Diamond\,\phi$



Guarded events

```
e
  any  t  where
    G.t.v
  then
    S.t.v.v'
  end
```

- t: parameters
- $G.t.v$: guard
- $S.t.v.v'$: action

- e.t is enabled when $G.t.v$ holds.
- Execution of e.t: v is updated according to the action $S.t.v.v'$.
- e.t corresponds to a formula act.(e.t).

Scheduled events (1/2)

e
any t **where**
...
during
$C.t.v$
upon
$F.t.v$
then
...
end

- $C.t.v$: coarse-schedule.
- $F.t.v$: fine-schedule.
- Healthiness condition:

$$C.t.v \land F.t.v \Rightarrow G.t.v$$

Liveness (Scheduling) Assumption

If $C.t.v$ holds infinitely long and $F.t.v$ holds infinitely often then eventually e.t is executed.

$$sched.(e.t) = \Box(\Box C \land \Box\Diamond F \Rightarrow \Diamond(F \land act.(e.t)))$$

Schedules vs. Fairness

e $\hat{=}$ **any** t **where** $G.t.v$ **during** $C.t.v$ **upon** $F.t.v$ **then** ... **end**

- Schedules are a generalisation of weak- and strong-fairness.
- Weak-fairness:
  If e is enabled infinitely long then e eventually occurs.
  - Let C be G and F be $\top$.
- Strong-fairness:
  If e is enabled infinitely often then e eventually occurs.
  - Let F be G and C be $\top$.

Scheduled events (2/2)

Conventions

e $\hat{=}$ **any** t **where** ... **during** $C.t.v$ **upon** $F.t.v$ **then** ... **end**

- Unscheduled events (without **during** and **upon**): C is $\bot$
- When only **during** is present (no **upon**), F is $\top$.
- When only **upon** is present (no **during**), C is $\top$.

Execution of Unit-B Models

$$ex.\mathbf{M} = saf.\mathbf{M} \land live.\mathbf{M} \quad (1)$$

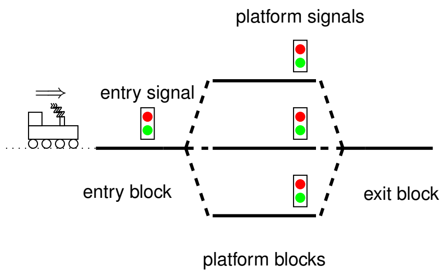
$$saf.\mathbf{M} = init \land \Box\, step.\mathbf{M} \quad (2)$$

$$step.\mathbf{M} = (\exists e.t \in \mathbf{M} \cdot act.(e.t)) \lor \text{SKIP} \quad (3)$$

$$live.\mathbf{M} = \forall e.t \in \mathbf{M} \cdot sched.(e.t) \quad (4)$$

$$sched.(e.t) = \Box(\Box C \land \Box\Diamond F \Rightarrow \Diamond(F \land act.(e.t))) \quad (5)$$

A Signal Control System (Recall)



SAF 1 There is at most one train on each block
LIVE 2 Each train in the network eventually leaves

Refinement strategy: Prioritise LIVE 2 first.

A Signal Control System. The Initial Model

- Focus on trains in the network
- Set *TRN* denotes the set of possible trains.
- Variable *trns* denotes the set of trains in the network.
- Event arrive models a train entering the network.
- Event depart models a train leaving the network.

```
arrive
  any t where
    t ∈ TRN
  then
    trns := trns ∪ {t}
  end
```

```
depart
  any t where
    t ∈ TRN
  during
    t ∈ trns
  then
    trns := trns \ {t}
  end
```


Execution and Properties

Execution and Properties

$\mathbf{M}$ satisfies ϕ if and only if $ex.\mathbf{M} \Rightarrow \phi$.

Safety Properties

- Invariance properties: (in LTL $\Box I$)
  - I holds for every reachable state.
  - Proved using the standard induction technique.
- Unless properties: P **un** Q
  - if P holds at some point then it continues to hold unless Q holds.
  - Prove: If for every event

    e $\widehat{=}$ **any** t **where** $G.t.v$ **during** ... **upon** ... **then** $S.t.v.v'$ **end**

    in **M**, we have

    $$P.v \land \neg Q.v \land G.t.v \land S.t.v.v' \quad \Rightarrow \quad P.v' \lor Q.v' \qquad \text{(UN)}$$

    then **M** satisfies P **un** Q.

Liveness Properties

- Progress properties $P \leadsto Q$.
- In LTL: $\Box(P \Rightarrow \Diamond Q)$
- Some important rules

$$
\begin{array}{rcll}
(P \Rightarrow Q) & \Rightarrow & (P \leadsto Q) & \text{(Implication)} \\
(P \leadsto Q) \land (Q \leadsto R) & \Rightarrow & (P \leadsto R) & \text{(Transitivity)} \\
(P \leadsto Q) & \Leftrightarrow & (P \land \neg Q \leadsto Q) & \text{(Split-Off-Skip)}
\end{array}
$$

A Signal Control System. The Initial Model

Properties

LIVE 2 Each train in the network eventually leaves

properties :
prg0_1 : $t \in trns \leadsto t \notin trns$

Note: Free-variables are universally quantified.

Transient Properties (1/3)

Definition

- Borrowed from UNITY.
- The basic tool for reasoning about progress properties.
- **tr** P states that always P is eventually falsified.
- In LTL: $\Box \Diamond \neg P$.
- Important properties:

$$\textbf{tr}\, P \quad = \quad \top \leadsto \neg P \quad = \quad P \leadsto \neg P$$

Transient Properties (2/3)

Theorem (Implementing tr)

if there exists an event

$e \mathrel{\widehat{=}}$ **any** t **where** $G.t.v$ **during** $C.t.v$ **upon** $F.t.v$ **then** $S.t.v.v'$ **end**

in **M** *such that*

$$
\begin{array}{rr}
\Box(P \Rightarrow C)\,, & \text{(SCH)} \\
C \leadsto F\,, & \text{(PRG)} \\
P.v \land C.t.v \land F.t.v \land G.t.v \land S.t.v.v' \;\Rightarrow\; \neg P.v' & \text{(NEG)}
\end{array}
$$

then **M** *satisfies* **tr** P.

- (SCH) corresponds to an invariance property.
- (PRG) is trivial when F is $\top$.
- (NEG) corresponds to a standard Hoare-triple.

Transient Properties (3/3)

A Sketch Proof

Consider $\textbf{tr}\, P \;=\; P \leadsto \neg P \;=\; \Box(P \Rightarrow \Diamond \neg P)$.

Proof (Sketch).

Assume P holds in some state, we prove $\Diamond \neg P$ by contradiction.

1. Assume $\Box P$.
2. From (SCH), we have $\Box C$,
3. together with (PRG), we have $\Box \Diamond F$.
4. Scheduling assumption ensures that e will eventually occur.
5. (NEG) guarantees that when e occurs, P is falsified.
6. We have a contradiction with the assumption from Step 1

□

A Signal Control System. The Initial Model

Properties

```
depart
  any  t  where
    t ∈ TRN
  during
    t ∈ trns
  then
    trns := trns \ {t}
  end
```

prg0_1 : $t \in trns \leadsto t \notin trns$

- prg0_1 is the same as $\textbf{tr}\; t \in trns$
- (SCH) is trivial.
- No fine-schedule (F is $\top$) hence (PRG) is trivial.
- The event falsifies $t \in trns$ (NEG)

Refinement

- Abstract systems can simulate behaviours of concrete systems.

$$ex.\textbf{cncM} \;\Rightarrow\; ex.\textbf{absM}$$

- Event-based reasoning.

(abs_)e $\widehat{=}$ **any** t **where** G **during** C **upon** F **then** S **end**

(cnc_)f $\widehat{=}$ **any** t **where** H **during** D **upon** E **then** R **end**

- Safety:
  - Guard strengthening: $H \Rightarrow G$
  - Action strengthening: $R \Rightarrow S$
- Liveness:
  - Liveness assumption strengthening.
  - Schedules weakening:

$$(\Box C \;\wedge\; \Box\Diamond F) \;\Rightarrow\; (\Box D \;\wedge\; \Box\Diamond E)$$

Schedules Weakening

Practical Rules

$$(\Box C \;\wedge\; \Box\Diamond F) \;\Rightarrow\; (\Box D \;\wedge\; \Box\Diamond E) \qquad \text{(REF\_LIVE)}$$

- Practical rules:
  - Coarse-schedule following

$$C \wedge F \leadsto D \qquad \text{(C\_FLW)}$$

  - Coarse-schedule stabilising

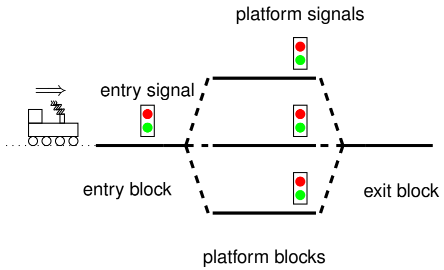
$$D \;\textbf{un}\; C \qquad \text{(C\_STB)}$$

  - Fine-schedule following

$$C \wedge F \leadsto E \qquad \text{(F\_FLW)}$$

A Signal Control System. The First Refinement

The State



- Introduce the network topology: BLK, $Entry$, PLF, $Exit$.
- Variable loc denotes location of trains in the network.

$$\text{inv1\_1} : loc \in trns \rightarrow BLK$$

A Signal Control System. The First Refinement

Refinement of depart

(abs_)depart
any t **where**
$t \in TRN$
during
$t \in trns$
then
$trns := trns \setminus \{t\}$
end

(cnc_)depart
any t **where**
$t \in trns \wedge loc.t = Exit$
during
$t \in trns \wedge loc.t = Exit$
then
$trns := trns \setminus \{t\}$
$loc := \{t\} \lhd loc$
end

- Guard and action strengthening are trivial.
- Coarse-schedule following (amongst others):

$$t \in trns \rightsquigarrow t \in trns \wedge loc.t = Exit \qquad (\text{prg1\_1})$$

A Signal Control System. The First Refinement

Refinement of depart

$t \in trns \rightsquigarrow t \in trns \wedge loc.t = Exit$

$\Leftrightarrow$ Put the negation of RHS in the LHS

$t \in trns \wedge loc.t \neq Exit \rightsquigarrow t \in trns \wedge loc.t = Exit$

$\Leftarrow$ Transitivity

$t \in trns \wedge loc.t \neq Exit \rightsquigarrow t \in trns \wedge loc.t \in PLF \wedge$
$t \in trns \wedge loc.t \in PLF \rightsquigarrow t \in trns \wedge loc.t = Exit$

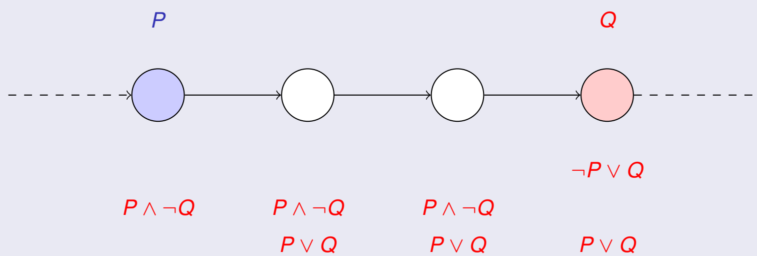
- The 1st condition is implemented by event movein (not shown)
- The 2nd condition is implemented by event moveout
- We need the ensure rule (next slide).

The Ensure Rule

Theorem (The ensure-rule)

For all state predicates p and q,

$$(P \;\mathbf{un}\; Q) \wedge (\mathbf{tr}\; P \wedge \neg Q) \Rightarrow (P \rightsquigarrow Q) \qquad \text{(ENS)}$$



A Signal Control System. The First Refinement

New Event moveout

$t \in trns \land loc.t \in PLF \;\leadsto\; t \in trns \land loc.t = Exit$

$\Leftarrow$ Ensure rule

$t \in trns \land loc.t \in PLF \;\textbf{un}\; t \in trns \land loc.t = Exit \;\land$
$(\textbf{tr}\,(t \in trns \land loc.t \in PLF) \land \neg(t \in trns \land loc.t = Exit))$

$\Leftrightarrow$ Logic

$\ldots \;\land\; (\textbf{tr}\; t \in trns \land loc.t \in PLF)$

```
moveout
  any t where
    t ∈ trns ∧ loc.t ∈ PLF
  during
    t ∈ trns ∧ loc.t ∈ PLF
  then
    loc.t := Exit
  end
```


A Signal Control System. The Second Refinement

The State

SAF 1 There is at most one train on each block

$$\forall t_1, t_2 \;\cdot\; t_1 \in trns \land t_2 \in trns \land loc.t_1 = loc.t_2 \;\Rightarrow\; t_1 = t_2$$

A Signal Control System. The Second Refinement

Refinement of moveout

```
(abs_)moveout
  any t where
    t ∈ trns ∧ loc.t ∈ PLF
  during
    t ∈ trns ∧ loc.t ∈ PLF
  then
    loc.t := Exit
  end
```

```
(cnc_)moveout
  any t where
    t ∈ trns ∧ loc.t ∈ PLF ∧
    Exit ∉ ran .loc
  during
    t ∈ trns ∧ loc.t ∈ PLF
  upon
    Exit ∉ ran .loc
  then
    loc.t := Exit
  end
```

- Neither weak- nor strong-fairness is satisfactory.
  - Weak-fairness requires *Exit* to be free infinitely long.
  - Strong-fairness is too strong assumption.

A Signal Control System. The Third Refinement

The State

- Introduce the signals *sgn*

inv3_1 : $sgn \in \{Entry\} \cup PLF \to COLOR$
inv3_2 : $\forall p \cdot p \in PLF \land sgn.p = GR \Rightarrow Exit \notin \mathrm{ran}\,.loc$
inv3_3 : $\forall p, q \cdot p, q \in PLF \land sgn.p = sgn.q = GR \Rightarrow p = q$

A Signal Control System. The Third Refinement

Refinement of moveout

(abs_)moveout
any t **where**
$t \in trns \land loc.t \in PLF \land$
$Exit \notin \mathrm{ran}\,.loc$
during
$t \in trns \land loc.t \in PLF$
upon
$Exit \notin \mathrm{ran}\,.loc$
then
$loc.t := Exit$
end

(cnc_)moveout
any t **where**
$t \in trns \land loc.t \in PLF \land$
$sgn.(loc.t) = GR$
during
$t \in trns \land loc.train \in PLF \land$
$sgn.(loc.t) = GR$
then
$loc.t := Exit$
$sgn.(loc.t) := RD$
end

Refinement requires to prove:

$$\textbf{tr}\ t \in trns \land loc.t \in PLF \land sgn.(loc.t) = RD\ . \qquad (\text{prg3\_5})$$

A Signal Control System. The Third Refinement

New Controller Event ctrl_platform

ctrl_platform
any p **where**
$p \in PLF \land p \in \mathrm{ran}\,.loc \land Exit \notin \mathrm{ran}\,.loc \land$
$(\forall q \cdot q \in PLF \Rightarrow sgn.q = RD)$
during
$p \in PLF \land p \in \mathrm{ran}\,.loc \land sgn.p = RD$
upon
$Exit \notin \mathrm{ran}(loc) \land (\forall q \cdot q \in PLF \land q \neq p \Rightarrow sgn.q = RD)$
then
$sgn.p := GR$
end

Summary

The Unit-B Modelling Method

- Guarded and scheduled events.
- Reasoning about liveness (progress) properties.
- Refinement preserving safety and liveness properties.
- Developments are guided by safety and liveness requirements.

Summary

Future Work

- Data refinement
- Decomposition / Composition
- Tool support

References I

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