Developing Control Systems in Event-B

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Outline

Event-B Modelling Method

- A modelling language for discrete transition systems.
- Mathematical language
  of first-order logic and some typed set theory.
- Incremental development process using refinement.
- Consistency of models: discharging proof obligations.
- Correct-by-construction systems.
- Supported by the RODIN Platform.

Event-B Models

Context	Machine
constants **carrier sets** **axioms**	**variables**: v **invariant**: $I(v)$ **events**: evt
Static part	Dynamic part

```
evt
  any t where
    G(t, v)
  then
    v := E(t, v)
  end
```

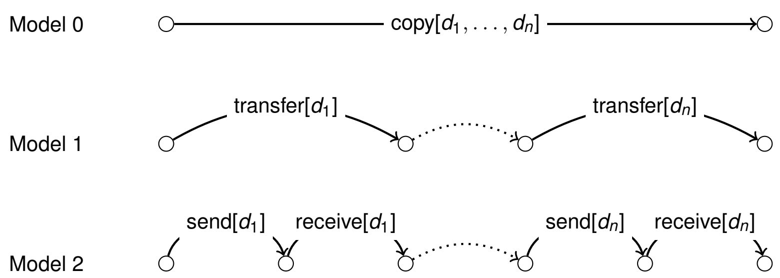
- t – the parameters.
- $G(t, v)$ – the guard: enable conditions.
- $v := E(t, v)$ – the action: v is assigned the value of $E(t, v)$.
- Initialisation: A special event without parameters and guards.

Consistency: Invariant establishment and preservation

Refinement

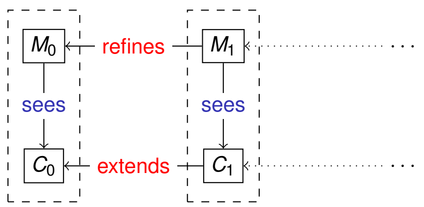
- A way to introduce more concrete details into the formal model.
- The concrete model must be consistent with the abstract model.
- Analogies with a microscope or a parachute.
- The view of the system gets more accurate.
- Allow to observe the system with a finer time grain.

Example. File Transfer



- Model 0: the file is copied in one-shot.
- Model 1: the file is transfered piece-by-piece.
- Model 2: each transfer is done via a pair of send/receive actions.

Event-B Refinement



Consistency: The concrete model only exhibits behaviours allowed by the abstract model.

- Event-wise reasoning:
  - Guard strengthening:
    concrete guards are stronger than abstract guards.
  - Simulation: The abstract event can simulate the concrete event.

Applications

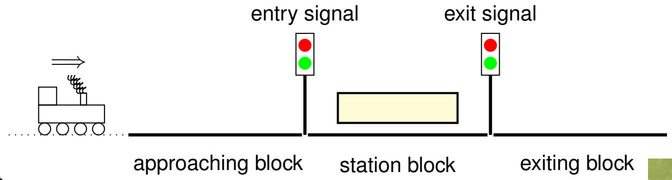
Event-B can be used to model:

- distributed systems,
- concurrent systems,
- sequential programs,
- electronic circuits,
- control systems,
- etc.

Outline

Train Control at a Stations

- Joint work with Simon Hudon.
- A station has a single track.
- The track is one way:
  - the train enters the station block via the approaching block.
  - the train exits the station block via the exiting block.
- There are two signals located at the two ends of the station.
- The signals turn to red automatically when a train passes by.
- The system controls when to turn the signals to green.



Environment

ENV 1	A train occupies no more than one block.
ENV 2	Each signal is either green or red.
ENV 3	Trains are assumed to stop at red signals.
ENV 4	The signals automatically change from green to red when some train passes by.

Safety Requirement

- The system guarantees that there is no collision between trains.

SAF 5	Two trains are not on the same block at the same time.

Train Schedule

- Each train is associated with a predefined route plan.
- The plan specifies either the train to stop or pass through.

FUN 6	Each train either stops or passes through according to a predefined route plan.

Sensors and Actuators

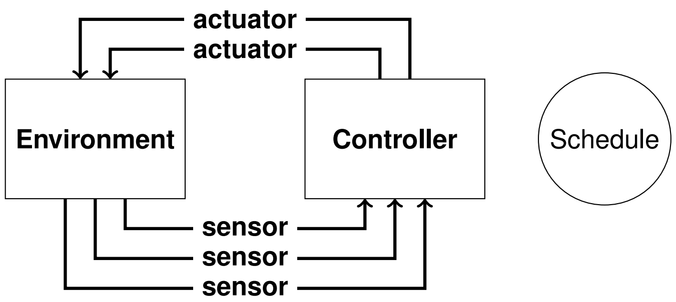
- There are sensors detecting if a block is occupied
- There are sensors detecting the status of the two signals.

ENV 7	The sensors always reflect the values of the corresponding physical components.

- The controller commands the signals via actuators.

ENV 8	For each signal, there is an actuator for the controller to turn it from red to green.

A Modelling Guideline for Developing Control Systems



Phase 1 Model the environment.
Phase 2 Model the actuators.
Phase 3 Model the sensors and the controller.
Phase 4 Model the schedule.

Phase 1. Environment
First Model. Trains and Tracks (1/3): The Context

carrier sets: $BLOCK, TRAIN$

constants: APP, STN, EXT

axioms:
axm0_1 : $\text{partition}(BLOCK, \{APP\}, \{STN\}, \{EXT\})$

- **axm0_1**: APP, STN, EXT are distinct blocks.

Phase 1. Environment

First Model. Trains and Tracks (2/3): The State

variables: *Trns*, *Loc*, *Occ*

init
  begin
    $Trns, Loc, Occ := \varnothing, \varnothing, \varnothing$
  end

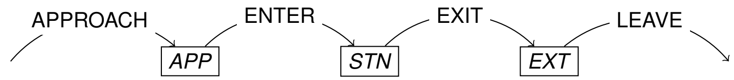
invariants:
  inv0_1 : $Trns \subseteq TRAIN$
  inv0_2 : $Loc \in Trns \to BLOCK$
  inv0_3 : $Occ = \mathrm{ran}(Loc)$
  inv0_4 : $\forall t_1, t_2 \cdot t_1 \in Trns \land t_2 \in Trains \land t_1 \neq t_2 \Rightarrow Loc(t_1) \neq Loc(t_2)$

- **inv0_1**: *Trns* is the set of "monitored" trains.
- **inv0_2–3**: Each monitored train occupies one block (ENV 1).
- **inv0_4**: No two trains are on the same block (SAF 5).

Phase 1. Environment

First Model. Trains and Tracks (3/3): The Events

- There are 4 events modelling the movements of trains.



APPROACH
  any t **where**
    $t \notin Trns$
    $APP \notin Occ$
  then
    $Trns := Trns \cup \{t\}$
    $Loc(t) := APP$
    $Occ := Occ \cup \{APP\}$
  end

ENTER
  any t **where**
    $t \in Trns$
    $Loc(t) = APP$
    $STN \notin Occ$
  then
    $Loc(t) := STN$
    $Occ := (Occ \cup \{STN\}) \setminus \{APP\}$
  end

- Guards guarantee safety properties.
- Events EXIT and LEAVE are similar.

Phase 1. Environment

Second Model. Signals

carrier sets: *LIGHT*

constants: *RED*, *GREEN*

axioms:
  axm1_1 : $\mathrm{partition}(LIGHT, \{RED\}, \{GREEN\})$

variables: $\ldots, Ent\_sgn, Ext\_sgn$

init
  begin
    ...
    $Ent\_sgn, Ext\_sgn := RED, RED$
  end

invariants:
  inv1_1 : $Ent\_sgn \in LIGHT$
  inv1_2 : $Ext\_sgn \in LIGHT$

- ENV 2: Signals are either red or green.

Phase 1. Environment

Second Model. Signals

(abs_)ENTER
  any t **where**
    ...
    $STN \notin Occ$
  then
    ...
  end

(cnc_)ENTER
  any t **where**
    ...
    $Ent\_sgn = GREEN$
  then
    ...
    $Ent\_sgn := RED$
  end

- ENV 3: Trains suppose to obey the signals.
- ENV 4: Signal changes from green to red automatically.
- Addition invariant (due to guard strengthening)

inv1_3 : $Ent\_sgn = GREEN \Rightarrow STN \notin Occ$

Phase 1. Environment

Second Model. Signals

CHANGE_ENTER_SIGNAL
when
$Ent\_sgn = RED$
$STN \notin Occ$
then
$Ent\_sgn := GREEN$
end

- Recall: invariant **inv1_3**

inv1_3 : $Ent\_sgn = GREEN \Rightarrow STN \notin Occ$

- Similar for events EXIT and CHANGE_EXIT_SIGNAL.

Phase 2. Actuators

variables: $\ldots, act\_ent\_sgn, act\_ext\_sgn$

invariants:
inv2_1 : $act\_ent\_sgn \in \mathrm{BOOL}$
inv2_2 : $act\_ext\_sgn \in \mathrm{BOOL}$

init
begin
...
$act\_ent\_sgn, act\_ext\_sgn := \mathrm{FALSE}, \mathrm{FALSE}$
end

Phase 2. Actuators

(abs_)CHANGE_ENTER_SIGNAL
when
$Ent\_sgn = RED$
$STN \notin Occ$
then
$Ent\_sgn := GREEN$
end

(cnc_)CHANGE_ENTER_SIGNAL
when
$act\_ent\_sgn = \mathrm{TRUE}$
then
$Ent\_sgn := GREEN$
$act\_ent\_sgn := \mathrm{FALSE}$
end

- Additional invariant

inv2_3 : $act\_ent\_sgn = \mathrm{TRUE} \Rightarrow Ent\_sgn = RED \land STN \notin Occ$

- ENV 8: The signals is changed accordingly to the actuators.
- Similar for event CHANGE_EXIT_SIGNAL.

Phase 2. Actuators

ctrl_change_enter_signal
when
$act\_ent\_sgn = \mathrm{FALSE}$
$Ent\_sgn = RED$
$STN \notin Occ$
then
$act\_ent\_sgn := \mathrm{TRUE}$
end

- Take into account the following invariant

inv2_3 : $act\_ent\_sgn = \mathrm{TRUE} \Rightarrow Ent\_sgn = RED \land STN \notin Occ$

- Similar for events ctrl_change_exit_signal and ctrl_change_both_signal.

Phase 3. Sensors and Controller

variables: $\ldots,$ $sen\_blk,$ $sen\_ent\_sgn,$ sen_ext_sgn

invariants:
inv3_1 : $sen\_blk = Occ$
inv3_2 : $sen\_ent\_sgn = Ent\_sgn$
inv3_3 : $sen\_ext\_sgn = Ext\_sgn$

init
begin
$\ldots$
$sen\_blk := \varnothing$
$sen\_ent\_sgn, sen\_ext\_sgn := RED, RED$
end

- *sen_blk*: Sensors detecting if a block is occupied.
- *sen_ent_sgn*, *sen_ext_sgn*: Sensors detecting status of signals.
- Invariants: Sensors reflect the status of components (ENV 7).

Phase 3. Sensors and Controller

- Additional assignment(s) in physical events set the value of the sensor appropriately.
- Example

ENTER
any t **where**
$\ldots$
then
$Loc(t) := STN$
$Occ := (Occ \cup \{STN\}) \setminus \{APP\}$
$Ent\_sgn := RED$
$sen\_blk := (sen\_blk \cup \{STN\}) \setminus \{APP\}$
$sen\_ent\_sgn := RED$
end

Phase 3. Sensors and Controller

(abs_)ctrl_change_enter_signal
when
$act\_ent\_sgn = \mathrm{FALSE}$
$Ent\_sgn = RED$
$STN \notin Occ$
then
$act\_ent\_sgn := \mathrm{TRUE}$
end

(cnc_)ctrl_change_enter_signal
when
$act\_ent\_sgn = \mathrm{FALSE}$
$sen\_ent\_sgn = RED$
$STN \notin sen\_blk$
then
$act\_ent\_sgn := \mathrm{TRUE}$
end

- Refinement is trivial with the invariants

inv3_1 : $sen\_blk = Occ$
inv3_2 : $sen\_ent\_sgn = Ent\_sgn$

Phase 4. Schedule

- FUN 6: Every train has some predefined route plan.

constants: $plan$

axioms:
axm4_1 : $plan \in TRAIN \rightarrow \mathrm{BOOL}$

- Plan of the train at the approaching block.

variables: a_plan

invariants:
inv4_1 : $\forall t \cdot t \in Trns \land$
$Loc(t) = APP \Rightarrow$
$a\_plan = plan(t)$

APPROACH
any t **where**
$\ldots$
then
$\ldots$
$a\_plan := plan(t)$
end

Phase 4. Schedule

```
ctrl_change_enter_signal
  when
    ...
    a_plan = TRUE
    APP ∈ sen_blk
  then
    ...
  end
```

```
ctrl_change_both_signal
  when
    ...
    a_plan = FALSE
    APP ∈ sen_blk
  then
    ...
  end
```

```
ctrl_change_exit_signal
  when
    ...
    STN ∈ sen_blk
  then
    ...
  end
```

- Schedule appropriately using the plan.
- Change the signals only when it is necessary.

Development Summary

Phase	Model	Requirement(s)
Phase 1	Model 0	ENV 1, SAF 5
	Model 1	ENV 2, ENV 3, ENV 4
Phase 2	Model 2	ENV 8
Phase 3	Model 3	ENV 7
Phase 4	Model 4	FUN 6

Summary. Event-B Modelling Method

- A modelling method for discrete transition systems.
- Mathematical language of first-order logic and set theory.
- Step-wise refinement to reduce development complexity.
- Correct by construction.
- Can be used to model a wide range of applications.

Summary. Developing Control System

- Start with model of the problem: the environment with various constraints.
- Step-by-step introduce:
  - Actuators (output of the controller).
  - Sensors (input of the controller) and the controller.
- Schedule the controller appropriately.
- Important features of the approach:
  - Safety properties are introduced early in terms of the environment: Safety properties are maintained by refinement.
  - Scheduling details in later phase of the development: Separation of concerns between safety properties and schedule.