Systems Design Guided by Progress Concerns

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Safety vs. Liveness

Safety Properties

- Something (bad) never happens.
- e.g. invariance properties

Liveness Properties

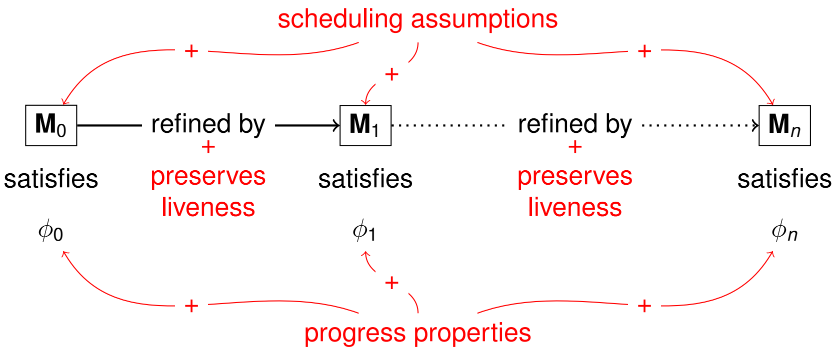
- Something (good) will happen
- e.g. termination, progress

- Liveness properties are essential.

Systems Development using Event-B

- $\phi_0, \phi_1, \ldots, \phi_n$: safety properties.

Unit-B = UNITY + Event-B



- Developments using Unit-B are guided by both safety and liveness requirements.

Traces and the Language of Temporal Logic

A trace σ is a (finite or infinite) sequence of states

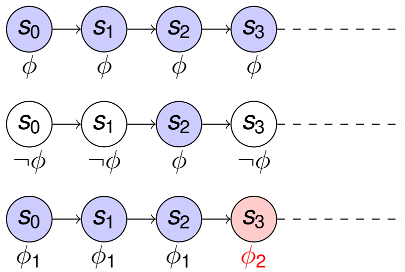
$$\sigma = s_0, s_1, s_2, s_3, \ldots$$

- A (basic) state formula P is any first-order logic formula,
- The basic formulae can be extended by combining the Boolean operators ($\neg, \wedge, \vee, \Rightarrow$) with temporal operators:

always: $\Box \phi$

eventually: $\Diamond \phi$

until: $\phi_1 \mathcal{U} \phi_2$



Unit-B Models. Guarded and Scheduled Events

e
  any t **where**
    $G.t.v$
  during
    $C.t.v$
  upon
    $F.t.v$
  then
    $S.t.v.v'$
  end

- Execution of e.t corresponds to a formula $act.(e.t)$.
- $C.t.v$: coarse-schedule.
- $F.t.v$: fine-schedule.
- Healthiness condition:

$$C.t.v \wedge F.t.v \Rightarrow G.t.v$$

Liveness (Scheduling) Assumption

If $C.t.v$ holds infinitely long and $F.t.v$ holds infinitely often then eventually e.t is executed when $F.t.v$ holds.

$$sched.(e.t) \quad = \quad \Box(\Box C \wedge \Box \Diamond F \Rightarrow \Diamond(F \wedge act.(e.t)))$$

Schedules vs. Fairness

e $\widehat{=}$ **any** t **where** $G.t.v$ **during** $C.t.v$ **upon** $F.t.v$ **then** … **end**

- Schedules are a generalisation of weak- and strong-fairness.
- Weak-fairness:
  If e is enabled infinitely long then e eventually occurs.
  - Let C be G and F be $\top$.
- Strong-fairness:
  If e is enabled infinitely often then e eventually occurs.
  - Let F be G and C be $\top$.

Conventions

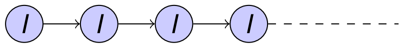
e $\widehat{=}$ **any** t **where** … **during** $C.t.v$ **upon** $F.t.v$ **then** … **end**

- Unscheduled events (without **during** and **upon**): C is $\bot$
- When only **during** is present (no **upon**), F is $\top$.
- When only **upon** is present (no **during**), C is $\top$.

Safety Properties

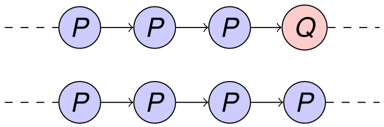
- Invariance properties:

$\Box I$

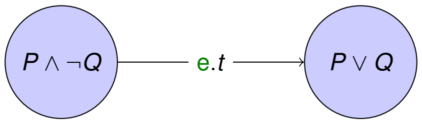


- Unless properties: P **un** Q

$\Box(P \Rightarrow P\,\mathcal{U}\,Q)$
or
$\Box(P \Rightarrow \Box P)$



- Prove: For every event e.t in **M**



Liveness Properties

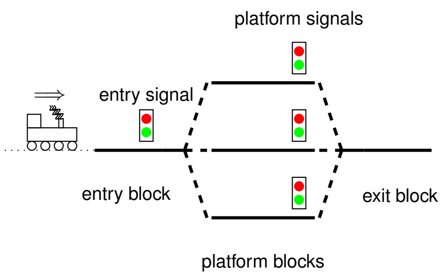
- Progress properties

$$P \leadsto Q \;\hat{=}\; \Box(P \Rightarrow \Diamond Q)$$

- Some important rules

$$(P \Rightarrow Q) \;\Rightarrow\; (P \leadsto Q) \qquad \text{(Implication)}$$
$$(P \leadsto Q) \wedge (Q \leadsto R) \;\Rightarrow\; (P \leadsto R) \qquad \text{(Transitivity)}$$

A Signal Control System



SAF 1 There is at most one train on each block

LIVE 2 Each train in the network eventually leaves

Refinement Strategy

Model 0 To model trains in the network, focus on LIVE 2
Ref. 1 To introduce the network topology
Ref. 2 To take into account SAF 1
Ref. 3 To introduce signals and derive a specification for the controller

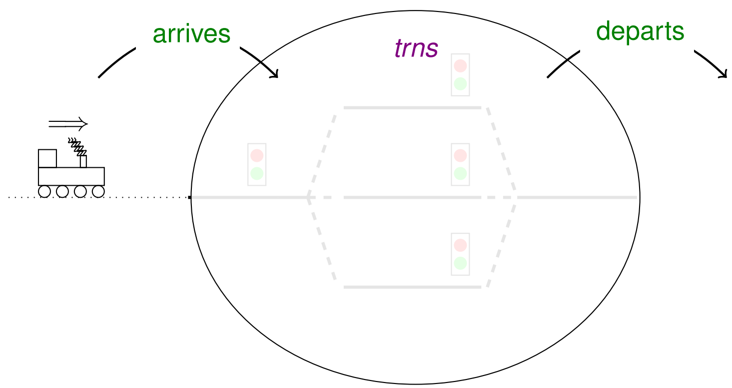
A Signal Control System. The Initial Model

Sketch

LIVE 2 Each train in the network eventually leaves

variables : $trns$

invariants :
$trns \subseteq TRN$



Transient Properties

Theorem (Implementing $P \leadsto \neg P$)

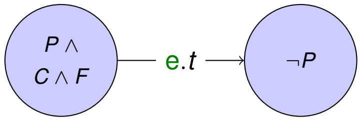
M *satisfies* $P \leadsto \neg P$ *if there exists an event in* **M**

$$e \mathrel{\widehat{=}} \textbf{any } t \textbf{ where } G.t.v \textbf{ during } C.t.v \textbf{ upon } F.t.v \textbf{ then } S.t.v.v' \textbf{ end}$$

such that

$$\square(P \Rightarrow C)\ , \qquad \text{(SCH)}$$

$$C \leadsto F\ , \qquad \text{(PRG)}$$



(NEG)

- Note: general progress properties can be proved using the *induction* or *ensure* rules.

A Signal Control System. The Initial Model

Properties

departs
- **any** t **where**
  - $t \in TRN$
- **during**
  - $t \in trns$
- **then**
  - $trns := trns \setminus \{t\}$
- **end**

prg0_1 : $t \in trns \leadsto t \notin trns$

- (SCH) is trivial.
- No fine-schedule (F is $\top$) hence (PRG) is trivial.
- The event falsifies $t \in trns$ (NEG)

Refinement

- Event-based reasoning.

$$\text{(abs\_)e} \mathrel{\widehat{=}} \textbf{any } t \textbf{ where } G \textbf{ during } C \textbf{ upon } F \textbf{ then } S \textbf{ end}$$

$$\text{(cnc\_)f} \mathrel{\widehat{=}} \textbf{any } t \textbf{ where } H \textbf{ during } D \textbf{ upon } E \textbf{ then } R \textbf{ end}$$

- Safety:
  - Guard strengthening: $H \Rightarrow G$
  - Action strengthening: $R \Rightarrow S$
- Liveness:
  - Scheduling assumptions strengthening.
  - Schedules weakening:

$$(\square C \wedge \square\lozenge F) \Rightarrow \lozenge(\square D \wedge \square\lozenge E) \qquad \text{(REF\_LIVE)}$$

Schedules Weakening

Practical Rules

$$(\square C \wedge \square\lozenge F) \Rightarrow \lozenge(\square D \wedge \square\lozenge E) \qquad \text{(REF\_LIVE)}$$

Practical rules

- Coarse-schedule following

$$C \wedge F \leadsto D \qquad \text{(C\_FLW)}$$

- Coarse-schedule stabilising

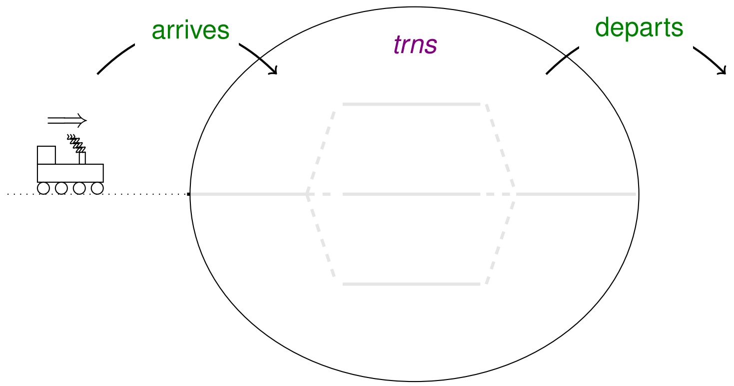
$$D \textbf{ un } \neg C \qquad \text{(C\_STB)}$$

- Fine-schedule following

$$C \wedge F \leadsto E \qquad \text{(F\_FLW)}$$

A Signal Control System. The First Refinement

The State



inv1_1 : $loc \in trns \rightarrow BLK$

A Signal Control System. The First Refinement

Refinement of departs

(abs_)departs
any t **where**
$t \in TRN$
during
$t \in trns$
then
$trns := trns \setminus \{t\}$
end

(cnc_)departs
any t **where**
$t \in trns \wedge loc.t = Exit$
during
$t \in trns \wedge loc.t = Exit$
then
$trns := trns \setminus \{t\}$
$loc := \{t\} \lhd loc$
end

- Guard and action strengthening are trivial.
- Coarse-schedule following (amongst others):

$$t \in trns \leadsto t \in trns \wedge loc.t = Exit \qquad (\text{prg1\_1})$$

A Signal Control System. The First Refinement

New Event moveout

moveout
any t **where**
$t \in trns \wedge loc.t \in PLF$
during
$t \in trns \wedge loc.t \in PLF$
then
$loc.t := Exit$
end

A Signal Control System. The Second Refinement

The State

SAF 1 There is at most one train on each block

invariants :
$\forall t_1, t_2 \cdot t_1 \in trns \wedge t_2 \in trns \wedge loc.t_1 = loc.t_2 \Rightarrow t_1 = t_2$

A Signal Control System. The Second Refinement

Refinement of moveout

(abs_)moveout
any t **where**
$t \in trns \wedge loc.t \in PLF$
during
$t \in trns \wedge loc.t \in PLF$
then
$loc.t := Exit$
end

(cnc_)moveout
any t **where**
$t \in trns \wedge loc.t \in PLF \wedge$
$Exit \notin \mathrm{ran}\,.loc$
during
$t \in trns \wedge loc.t \in PLF$
upon
$Exit \notin \mathrm{ran}\,.loc$
then
$loc.t := Exit$
end

- Neither weak- nor strong-fairness is satisfactory.
  - Weak-fairness requires *Exit* to be free infinitely long.
  - Strong-fairness is too strong assumption.

Summary

The Unit-B Modelling Method

- Guarded and scheduled events.
- Reasoning about liveness (progress) properties.
- Refinement preserving safety and liveness properties.
- Developments are guided by safety and liveness requirements.

Future Work

- Decomposition / Composition
- Tool support

Refinement

The UNITY way vs. the Event-B way

- UNITY: Refines the formulae.

$$\overbrace{\phi \Leftarrow \phi_1 \Leftarrow \ldots \Leftarrow \phi_n}^{\text{Refinement}} \underbrace{\;\dashv\; \mathbf{M}}_{\text{Translation}}$$

  - Cons: Hard to understand the choice of refinement.

- Event-B: Refines transition systems.

$$\underbrace{\phi \;\dashv\; \mathbf{M}_0}_{\text{Verification}} \overbrace{\text{ refined by } \mathbf{M}_1 \ldots \text{ refined by } \mathbf{M}}^{\text{Refinement}}$$

  - Cons: No support for liveness properties.

Execution of Unit-B Models

$$ex.\mathbf{M} = saf.\mathbf{M} \wedge live.\mathbf{M} \quad (1)$$

$$saf.\mathbf{M} = init.\mathbf{M} \wedge \Box\, step.\mathbf{M} \quad (2)$$

$$step.\mathbf{M} = (\exists e.t \in \mathbf{M} \cdot act.(e.t)) \vee \text{SKIP} \quad (3)$$

$$live.\mathbf{M} = \forall e.t \in \mathbf{M} \cdot sched.(e.t) \quad (4)$$

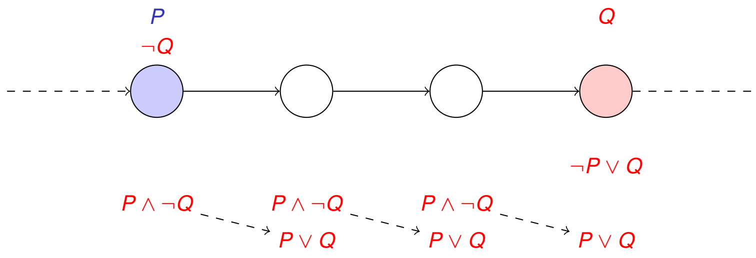
$$sched.(e.t) = \Box(\Box C \wedge \Box\Diamond F \Rightarrow \Diamond(F \wedge act.(e.t))) \quad (5)$$

The Ensure Rule

Theorem (The ensure-rule)

For all state predicates p and q,

$$(P \ \mathbf{un}\ Q) \ \wedge \ ((P \wedge \neg Q) \leadsto (\neg P \vee Q)) \quad \Rightarrow \quad (P \leadsto Q) \qquad \text{(ENS)}$$



The specification of the controller

ctrl_platform
any p **where**
$p \in PLF \wedge p \in \mathrm{ran}\ .loc \wedge Exit \notin \mathrm{ran}\ .loc \wedge$
$\forall q \cdot q \in PLF \Rightarrow sgn.q = RD$
during
$p \in PLF \wedge p \in \mathrm{ran}\ .loc \wedge sgn.p = RD$
upon
$Exit \notin \mathrm{ran}(loc) \wedge \forall q \cdot q \in PLF \wedge q \neq p \Rightarrow sgn.q = RD$
then
$sgn.p := GR$
end