The Weak Nuclear Force

Question 1:

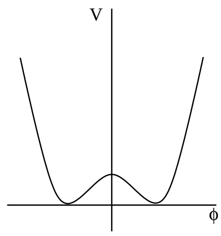
Consider a *real* scalar field, ϕ, with potential

$$V = \lambda(\phi^2 - m^2)^2$$

What symmetry does this theory possess (if it's not obvious sketch the potential). Find the vev of ϕ at the minimum and value of the potential in the vacuum. Are there Goldstone bosons in this example of the higgs mechanism?

Answer 1:

The potential looks like:



The symmetry is the discrete one: $\phi \to -\phi$.

For the minimum we require

$$\frac{dV}{d\phi} = 0 = \lambda(4\phi^3 - 4m^2\phi)$$

So at the minimum

$$\phi = \pm m$$

The value of the potential there is

$$V = \lambda(m^2 - m^2)^2 = 0$$

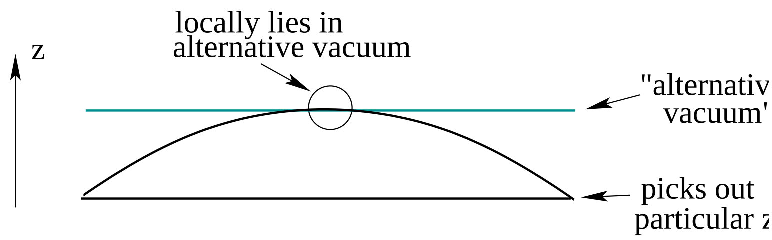
This potential does not generate Goldstone bosons - there is no continuous set of vacua. One can not distort the field in space without there being a direct cost in energy from the potential which corresponds to a mass.

Question 2:

Explain why a straight piece of string breaks translational invariance. Explain why waves on a string are Goldstone bosons.

Answer 2:

Translational invariance tells us that all points related by a translation along a particular axis are equivalent. If you lie a staright piece of string at some fixed value of z (see figure) then that value of z is different and translational invariance is broken. Note this is a spontaneous breaking because making a translation moves the string to a new value of z ie a new "vacuum" - the energy of the configuration matches that of the original.



Consider now some wave excitation on the string. As shown in the figure, locally the string lies in a new vacuum state. There is no energy cost to placing the string in this new position.. the energy cost comes from the derivatives as the string changes its values of z. This makes it a Goldstone boson.

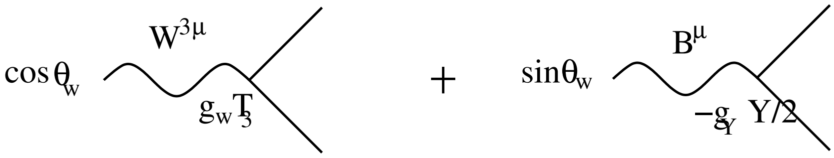
Question 3:

Show that the coupling of the Z to a fermion is of the form

$$\frac{e}{\cos\theta_w \sin\theta_w}(T^3 - \sin^2\theta_w Q)$$

Answer 3:

Each chiral fermion has a contribution to its coupling to the Z one from $W^{3\mu}$ and one from B^μ



$$g_Z = g_w \cos\theta_w T_3 - g_Y \sin\theta_w \frac{Y}{2}$$

We now use the relations

$$Q = T_3 + Y/2$$

and

$$e = g_w \sin\theta_w = g_y \cos\theta_w$$

So

$$g_z = \frac{e}{\sin\theta_w}\cos\theta_w T_3 - \frac{e}{\cos\theta_w}\sin\theta_w(Q - T_3)$$

ie

$$\begin{aligned} g_z &= \tfrac{e}{\sin\theta_w \cos\theta_w}\left(\cos^2\theta_w T_3 - \sin^2\theta_w Q + \sin^2\theta_w T_3\right) \\ &= \tfrac{e}{\cos\theta_w \sin\theta_w}(T^3 - \sin^2\theta_w Q) \end{aligned}$$

Question 4: (hard)

This question explains why Cabibbo mixing only happens in the down quark sector.

Write down the most general possible form of the mass matrices that the higgs could generate in the u, c and s, d sectors being careful to include chirality labels.

What flavour rotations can you make on the quark doublets? Which of these will effect the weak eigenstate basis?

Given that in general one needs two matrices, U, V to diagonalize an arbitrary matrix ($M_{diag} = VMU^\dagger$) explain why Cabibbo mixing is only in the down quark sector.

Answer 4:

In general we can have in the up sector

$$(u_L, c_L)^\dagger \begin{pmatrix} m_{uu} & m_{uc} \\ m_{cu} & m_{cc} \end{pmatrix} \begin{pmatrix} u_R \\ c_R \end{pmatrix}$$

The massless Dirac equation does not mix left and right handed spinors so is invariant to global SU(2) transformations

$$\begin{pmatrix} u_R \\ c_R \end{pmatrix} \to U \begin{pmatrix} u_R \\ c_R \end{pmatrix} = e^{i\theta_R^a T^a} \begin{pmatrix} u_R \\ c_R \end{pmatrix}, \qquad \begin{pmatrix} u_L \\ c_L \end{pmatrix} \to V \begin{pmatrix} u_L \\ c_L \end{pmatrix} = e^{i\theta_L^a T^a} \begin{pmatrix} u_L \\ c_L \end{pmatrix}$$

Note $\theta_L \neq \theta_R$. The separate transformations on left and right spinors allows us to write the mass term as

$$(u_L, c_L)^\dagger V^\dagger V \begin{pmatrix} m_{uu} & m_{uc} \\ m_{cu} & m_{cc} \end{pmatrix} U^\dagger U \begin{pmatrix} u_R \\ c_R \end{pmatrix} = (u'_L, c'_L)^\dagger V \begin{pmatrix} m_{uu} & m_{uc} \\ m_{cu} & m_{cc} \end{pmatrix} U^\dagger \begin{pmatrix} u'_R \\ c'_R \end{pmatrix}$$

Where the primed fields are those after an SU(2) transformation that have the usual kinetic terms in the Dirac equation. We see that we have enough freedom to diagonalize the matrix.

One would imagine everything would be the same in the down sector where the mass matrix is

$$(d_L, s_L)^\dagger \begin{pmatrix} m_{dd} & m_{ds} \\ m_{sd} & m_{ss} \end{pmatrix} \begin{pmatrix} d_R \\ s_R \end{pmatrix}$$

The crucial point though is that u_L and d_L are part of the same SU(2) gauge doublet for the weak force. We must treat them as equivalent particles and anything we do to one we must do to the other. We can not therefore make an arbitrary free rotation on d_L so we do not have enough freedom to diagonalize the down sectors mass matrix. There is a difference between the gauge and mass eigenstates in the down sector.