Relativistic Quantum Mechanics

Question 1:

What is a mass of 1 GeV in kg? What is a cross section of 1 GeV^{-2} in barns?

Question 2:

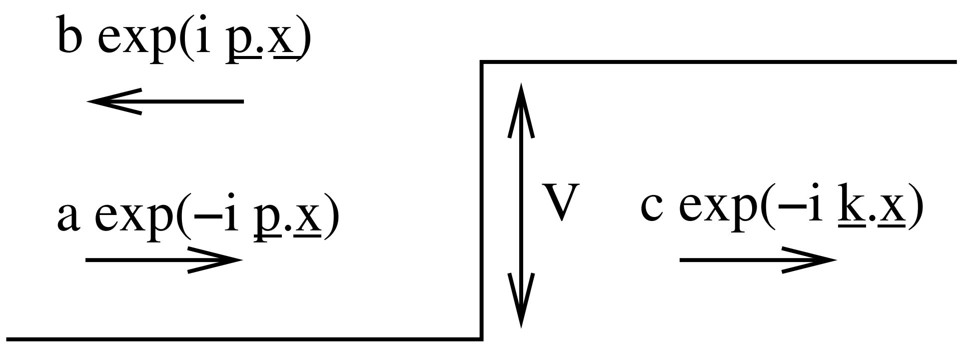
Derive the expressions for probability density in the Klein Gordon and Dirac equations.

Question 3:

How would you introduce a potential into the Klein Gordon equation? Think about dimensions and the non-relativistic limit!

Question 4: (involved)

Consider the wave incident on a potential step shown below.



Show that if $V > m + E_p$, where $E_p = \sqrt{v_p^2 + m^2}$ then one cannot avoid using the negative square root $v_k = -\sqrt{(E_p - V)^2 + m^2}$ and getting negative currents and densities. Hint: use the fact that $\phi(x)$ and $\partial\phi(x)/\partial x$ are continuous at $x = 0$, and ensure that the group velocity $v_g = \partial E/\partial k$ is positive for $x > 0$. Interpret the solution.

Question 5:

Prove that the matrices $\vec{\alpha}$ and β in the Dirac equation are traceless with eigenvalues ± 1.Hence argue that they must be even dimensional.

Question 6:

Show that for the Dirac equation solution spinors

$$u^\dagger(r,p)\; u(s,p) \;=\; v^\dagger(r,p)\; v(s,p) \;=\; 2E\delta^{rs}$$

Question 7:

Show using the properties of $\vec{\alpha}$ and β that the γ^μ must satisfy the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

Question 8:

Show that

$$\gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0$$