SEMESTER 2 EXAMINATION 2011/12

PARTICLE PHYSICS

Duration: 120 MINS

VERY IMPORTANT NOTE

Section A answers MUST BE in a separate blue answer book. If any blue answer booklets contain work for both Section A and B questions - the latter set of answers WILL NOT BE MARKED.

Answer ***all*** *questions in* ***Section A*** *and two* ***and only two*** *questions in* ***Section B****.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A Sheet of Physical Constants will be provided with this examination paper.

An outline marking scheme is shown in brackets to the right of each question.

Only university approved calculators may be used.

Section A

A1. The free Dirac equation is given by

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

where m is the particle's mass and γ^μ are the Dirac gamma matrices. Show that for the equation to be consistent with the Theory of Relativity the gamma matrices must satisfy the algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}.$$

(Hint: relate the Dirac gamma matrices to the Dirac alpha and beta matrices.) [4]

A2. Write an explicit expression for the ratio

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

assuming only photon mediation. [4]

A3. List nine colour states that can be built from a quark and an anti-quark and identify the linear combinations that transform as 'singlets' and 'octets' under $SU(3)$ colour transformations. Explain what they refer to. [4]

A4. Draw four potentially interesting Feynman diagrams corresponding to Standard Model Higgs boson production at the Large Hadron Collider (LHC). [4]

A5. Explain what particle physicists mean by the word 'regularisation'. [2]

A6. Explain what particle physicists mean by the term 'transverse momentum'. [2]

Section B

B1.

(a) Justify the Klein-Gordon equation

$$(\partial_\mu \partial^\mu + m^2)\phi = 0$$

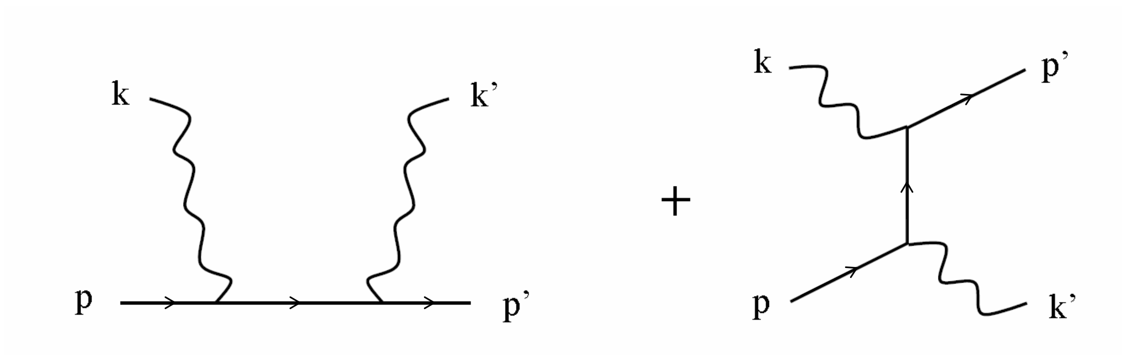
for a wave-function $\phi \equiv \phi(x) \equiv \phi(t, \mathbf{x})$ representing a relativistic scalar particle of mass m. [4]

(b) For plane-wave solutions in co-ordinate space, show that the corresponding energy eigenvalues are not positive definite. [4]

(c) Derive the continuity equation corresponding to the Klein-Gordon equation and prove that the probability density is not positive definite either. [8]

(d) Explain the Feynman-Stuckelberg interpretation of the negative energy solutions. Sketch a diagram to illustrate a physical process where this is manifest. [4]

B2. Compton scattering (the scattering of an electron and a photon) is described by the two Feynman diagrams



(a) We normally assume that the external photons satisfy the free Maxwell’s equation

$$\partial^{\nu}\partial_{\nu}A^{\mu}(x) = 0.$$

Show that there are solutions of the form $A^{\mu}(x) = \epsilon^{\mu}(q)e^{-iq^{\lambda}x_{\lambda}}$ where x^{λ} is the position four-vector and ϵ^{μ} is any polarisation vector. [2]

(b) Similarly the external electron fields satisfy the free Dirac equation

$$(i\gamma^{\mu}\partial_{\mu} - m)\psi(x) = 0.$$

There are solutions of the form $\psi(x) = u(p)e^{-ip^{\lambda}x_{\lambda}}$. Show that $u(p)$ satisfies

$$(\gamma^{\mu}p_{\mu} - m)u(p) = 0.$$

[1]

(c) The amplitude at leading order in perturbation theory for the scattering of an electron of charge q from a state a to a state c as the result of the interaction with a $U(1)$ gauge potential $A^\mu(x)$ is

$$\mathcal{M}_{ac} = -i \int J^\mu_{ac}(x)\, A_\mu(x)\, d^4x \qquad \text{where} \qquad J^\mu_{ac}(x) = iq(\bar{\psi}_c(x)\gamma^\mu\psi_a(x)).$$

The internal (or virtual) electron wave function $\psi_{\rm int}(x)$ may be found by solving the Dirac equation in the presence of the photon gauge field

$$(i\gamma^\mu\partial_\mu - m)\psi_{\rm int}(x) = q\gamma_\mu A^\mu_{ext}(y)\psi_{ext}(y),$$

where the subscript ext corresponds to an external (or real) particle at the vertex.

Using these results show that the contribution to the scattering amplitude from the first diagram above is given by

$$-i\mathcal{M}_{ac} \;=\; \bar{u}(p')\left(\epsilon^*_\nu(k')\;\; iq\gamma^\nu\;\; \tfrac{i(\not{p}+\not{k})+m}{(p'+k')^2-m^2}\;\; iq\gamma^\mu\;\; \epsilon_\mu(k)\right) u(p)$$
$$(2\pi)^4\delta^4(p+k-p'-k').$$

[10]

(d) Explain how this expression can be associated with a set of Feynman rules for the external electron and photon states, the vertices and the internal fermion propagator. [4]

(e) Using the Feynman rules derived in (d) write down the contribution to the scattering amplitude for the second diagram above. [3]

TURN OVER

B3. The Lagrangian for a complex scalar field is given by

$$\mathcal{L}_s = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) + \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2$$

where

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

and μ^2 and λ are constants.

(a) Show that, if we parameterise the four independent components of the complex doublet field ϕ as

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix},$$

then $\mathcal{L}_s$ is invariant under $O(4)$ rotations, i.e.,

$$\phi_i \to \phi_i' = \hat{r}_{ij}\phi_j, \qquad \text{with} \qquad i = 1, ...4,$$

where $\hat{r}$ is the four-dimensional rotation matrix

$$\hat{r}^T \hat{r} = \hat{r}\hat{r}^T = \mathbb{I}.$$

(Here the symbol T refers to the transpose of a matrix.) [6]

(b) Show that, if we write

$$\pi = (\phi_1, \phi_2, \phi_4), \qquad \sigma = \phi_3,$$

then the Lagrangian can be rewritten as

$$\mathcal{L}_s = \frac{1}{2}[(\partial_\mu \pi)^2 + (\partial_\mu \sigma)^2] + \frac{\mu^2}{2}(\pi^2 + \sigma^2) - \frac{\lambda}{4}(\pi^2 + \sigma^2)^2.$$

[3]

(c) For spontaneous symmetry breaking, we have

$$\phi_3 = \sigma = v + H, \qquad \text{with} \qquad v^2 = \frac{\mu^2}{\lambda}.$$

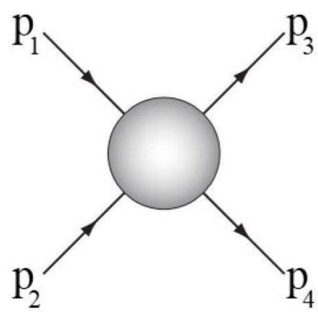
Write the Lagrangian in terms of H and π. What is the H field ? Extract the H mass. [7]

(d) Finally find the $H\pi^+\pi^-$ and Hzz couplings, where

$$\pi^+ = \frac{1}{\sqrt{2}}(\pi_1 - i\pi_2) \qquad \text{and} \qquad z = \pi_3.$$

What are the $\pi^\pm$ and z fields ? [4]

B4. Consider a two-to-two particle scattering



where the four-momenta of the particles $p_1^\mu, p_2^\mu, p_3^\mu$ and p_4^μ are such that

$$p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu$$

and

$$p_1^2 = m_1^2, \quad p_2^2 = m_2^2, \quad p_3^2 = m_3^2, \quad p_4^2 = m_4^2,$$

where m_1, m_2, m_3 and m_4 are the particle masses.

(a) Define the Mandelstam invariants s, t and u. [3]

(b) Prove that

$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2.$$

[4]

(c) The letters s, t and u are also used in the terms s-channel, t-channel and u-channel. These channels represent Feynman diagrams of different possible scattering events where the interaction involves the exchange of an intermediate particle whose squared four-momentum equals s, t or u. Sketch these Feynman diagrams. [3]

(d) The differential cross section for this process is given by the formula

$$d\sigma = \frac{|\overline{\mathcal{M}}|^2}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} d\Phi_2(p_1, p_2, p_3, p_4)$$

where $\mathcal{M}$ is the scattering matrix element and $d\Phi_2$ is the Lorentz Invariant Phase Space (LIPS) for a two-body final state. Write the corresponding expression for the LIPS term and explain the meaning of each term in it. [4]

(e) The above expression for the cross section is valid in any inertial reference frame. Prove that, in the centre-of-mass frame (i.e., $\mathbf{p}_1 = -\mathbf{p}_2$), one has

$$\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2} = |\mathbf{p}_1| \sqrt{s} = |\mathbf{p}_2| \sqrt{s}$$

for the so-called flux factor. [6]

END OF PAPER