UNIVERSITY OF SOUTHAMPTON PHYS6011W1

SEMESTER 2 EXAMINATION 2010/11

PARTICLE PHYSICS

Duration: 120 MINS

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*Answer **all** questions in **Section A** and two **and only two** questions in **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A Sheet of Physical Constants will be provided with this examination paper.

An outline marking scheme is shown in brackets to the right of each question.

Only university approved calculators may be used.

Section A

A1. Starting from the massless Klein-Gordon equation

$$\partial_\mu \partial^\mu \phi = 0,$$

where ϕ represents the wavefunction of the relevant state, show that the probability density current

$$J^\mu = i(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*)$$

satisfies the continuity equation

$$\partial_\mu J^\mu = 0.$$

[4]

A2. Explain Dirac's interpretation of negative energy states amongst the solutions of the Dirac equation. [4]

A3. Draw all eight Feynman diagrams for the following process

$$e^+(1)e^-(2) \rightarrow q(3)\overline{q}(4)q(5)\overline{q}(6)$$

where the numbers label particle momenta p_i and spin s_i ($i = 1...6$). Include only diagrams of order $\alpha^2\alpha_s^2$, where α and α_s are the electromagnetic and strong coupling 'constants', respectively, and ignore Z propagators. Note down the relative signs among all diagrams. Justify the relative signs in terms of Fermi-Dirac statistics. [5]

A4. Explain why in the Standard Model the decay of the Z particle into two Higgs bosons, $Z \to HH$, is forbidden by angular momentum conservation and Bose-Einstein statistics. [3]

A5. Briefly explain what is meant by renormalisation. Sketch the one-loop diagrams in QED which require renormalisation. [2]

A6. What is synchrotron radiation ? [2]

Section B

B1. The Dirac spinor associated with the positive energy solution can be written in momentum space as

$$u(p, \pm) \propto \begin{pmatrix} 1 \\ \frac{\vec{\sigma}\cdot\mathbf{p}}{E+m} \end{pmatrix} \chi_\pm$$

where $(\vec{\sigma} \cdot \hat{\mathbf{p}})\chi_\pm = \pm\chi_\pm$ with $\hat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}|$.

(a) Show that the left-handed (L) and right-handed (R) spinors given by

$$u_L(p) = \frac{1}{2}(1 - \gamma_5)u(p, -) \qquad \text{and} \qquad u_R(p) = \frac{1}{2}(1 + \gamma_5)u(p, +)$$

are eigenstates of the helicity operator $\lambda = \mathbf{s}\cdot\hat{\mathbf{p}}$ in the *massless* limit, where the spin operator is of the form

$$\mathbf{s} = \frac{1}{2}\begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}.$$

(Assume

$$\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.)$$

[12]

(b) The two operators $P_R = \frac{1}{2}(1 + \gamma_5)$ and $P_L = \frac{1}{2}(1 - \gamma_5)$ are the 'helicity projection operators'. The other two combinations

$$\frac{1}{2}(1 + \gamma_5)u(p, -) \qquad \text{and} \qquad \frac{1}{2}(1 - \gamma_5)u(p, +)$$

are identically zero in the massless limit. Explain the physical meaning of this result. [2]

(c) Sketch in a diagram the relative orientation of $\mathbf{s}$ and $\mathbf{p}$ for spin $1/2$ fermions which are right-handed and left-handed. [2]

(d) Show the relation between the two spinors $u_L(p)$ and $u_R(p)$ under parity transformation and explain under which circumstances an interaction could violate parity. [4]

B2. (a) Explain why QCD has an invariance under the action of unitary 3×3 matrices. [4]

(b) How many degrees of freedom do the 3×3 unitary matrices have ? [2]

(c) Explain what the algebra

$$3 \otimes \bar{3} = 8 \oplus 1$$

means when related to a bound state of a quark and an anti-quark. [6]

(d) What is the significance of the product in (c) for the number of physical gluons ? [2]

(e) Explain what a di-quark is and illustrate its colour state in relation to the above algebra. [4]

(f) Explain whether it would be possible for there to be a colour neutral particle made of four quarks and one anti-quark. [2]

B3. (a) Write down the SU(2)$_W$× U(1)$_Y$ Higgs potential in the Standard Model. Pay attention to the signs of the coefficients in the terms involved and define the Higgs field carefully. [6]

(b) In principle, one can also add to the potential an SU(2)$_W$× U(1)$_Y$ quartic invariant of the form

$$V'(\phi) = \lambda' \sum_{a,b} (\phi^\dagger \sigma^a \sigma^b \phi)(\phi^\dagger \sigma^a \sigma^b \phi)$$

where $\vec{\sigma}$ represents the usual Pauli matrices. Show that this quartic term can be reduced to the form

$$(\phi^\dagger \phi)^2$$

and comment on the significance of this.

You can use the relation

$$\sum_a (\sigma^a)_{ij}(\sigma^a)_{kl} = 2(\delta_{jk}\delta_{il} - \frac{1}{2}\delta_{ij}\delta_{kl}).$$

[9]

(c) Explain why there is only one physical Higgs state in the Standard Model particle spectrum. [5]

B4. (a) At what accelator facility and by what experiments was the top (anti-)quark discovered ? What initial particles were collided in order to produce it ? [2]

(b) Sketch a Feynman diagram describing the production and decay mechanisms exploited in the discovery. Label each particle at each stage of the process. [7]

(c) What are the approximate branching ratios of the unstable particles involved in the discovery channel ? Hence estimate the fraction of all top/anti-top events that decay via this channel. [4]

(d) Amongst the decay products of a top (anti-)quark one finds a bottom (anti-)quark. What is special about this object which renders the jet that originates from it more easily identifiable in a detector with respect to those produced by lighter (anti-)quarks and gluons ? [3]

(e) How was the top (anti-)quark mass estimated from the data ? [2]

(f) What is meant by the Yukawa coupling of a top (anti-)quark and how is this quantity related to its mass ? [2]

END OF PAPER