SEMESTER 2 EXAMINATION 2009/10

PARTICLE PHYSICS

Duration: 120 MINS

*Answer **all** questions in **Section A** and two **and only two** questions in **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A Sheet of Physical Constants will be provided with this examination paper.

Note that throughout this paper natural units are assumed in which $c = \hbar = 1$ except where SI units are explicitly used.

An outline marking scheme is shown in brackets to the right of each question.

Only university approved calculators may be used.

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Section A

A1. The Dirac equation is given by

$$(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0$$

where m is the particle's mass and γ^{μ} are the Dirac gamma matrices. Show that for the equation to be consistent with Relativity the gamma matrices must satisfy the algebra

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu}$$

[4]

A2. Explain briefly why the electromagnetic coupling α_{EM} runs with energy scale in Quantum Electrodynamics. [4]

A3. The LHC is designed to run with 2808 bunches and 10^{11} protons per bunch with a beam energy of 7 TeV. What is the total energy (in Joules) stored in one beam?

[2]

The world's largest passenger vessel 'Oasis of the Seas' weighs about 100,000 tonnes (a tonne is 1000kg). How fast would it need to travel in km/hr to have the same energy as one beam in the LHC? [2]

A4. There are three colours of a quark. Explain why combinations of two of the quarks split into two groupings one with six elements and one with three as expressed by the algebra

$$3 \otimes 3 = 6 \oplus \bar{3}$$

[4]

A5. The potential for a complex scalar particle with wave function ϕ, takes the form

$$V = -m^2|\phi|^2 + \lambda|\phi|^4$$

Explain what symmetry is present in this model, what symmetry breaking occurs and why there is a massless particle in the spectrum. [4]

TURN OVER

Section B

B1. The one dimensional Klein Gordon wave equation for a relativistic scalar particle of mass m in a potential V is given by

$$\left[-\left(i\frac{\partial}{\partial t}-V\right)^2-\frac{\partial}{\partial x^2}+m^2\right]\phi=0$$

(a) Explain how the Klein Gordon equation reproduces the standard relativistic energy momentum relation for a free particle (ie with $V=0$). [3]

(b) Show that when the particle is moving slowly in a potential the energy relation reduces to the expected non-relativistic form. [3]

(c) Starting from the Klein Gordon equation, show that the conserved probability density for the particle in the potential V is given by

$$\rho=\phi^*(i\frac{\partial}{\partial t}-V)\phi+\phi(-i\frac{\partial}{\partial t}-V)\phi^*$$

[8]

(d) Particles are fired at a potential step of height V_0 to the right of $x=0$. The incoming wave is of the form

$$\phi_{\text{inc}}=Ae^{-i(E_pt-px)}$$

A reflected wave travelling away from the barrier to the left is observed with value

$$\phi_{\rm ref} = A\frac{(p+k)}{(p-k)}e^{-i(E_p t + px)}$$

where k is the positive root of

$$k = \sqrt{(E_p - V)^2 - m^2}$$

What do you infer from the ratio of the amplitudes of the incoming and outgoing waves? [2]

(e) To be a consistent solution of the Klein Gordon equation there must be a wave to the right of the barrier of the form

$$\phi_{\rm bar} = \frac{2p}{p+k}Ae^{-i(E_p t + kx)}$$

What is the sign of the probability density of the particles in the barrier? [2]

(f) Interpret what is occuring at the barrier [2]

B2. At high energies we may neglect the mass of the electron. The four-component, left handed chiral electron wave function e_L then satisfies the Dirac equation

$$i\partial^\mu \gamma_\mu e_L = 0$$

where γ^μ are Dirac gamma matrices.

(a) This Dirac equation has a global U(1) symmetry - identify how the wave function transforms under that symmetry. [2]

(b) Show that if the symmetry is promoted to a local or gauge symmetry that one must introduce a massless gauge particle. Introduce the relevant gauge particle's wave function, A^μ, and show how it must transform for the equations to be invariant under the symmetry transformations. Explicitly show that invariance and why a mass term would break it. [12]

(c) In the electroweak theory the left handed electron is part of a doublet with the left handed neutrino.

$$\psi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$$

The generators of the full symmetry transformations on the doublet are

$$T^1 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad T^2 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$T^3 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad T^4 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

T^1 generates the U(1) hypercharge symmetry. The remaining three generators give rise to an SU(2) symmetry.

Explain why gauging those SU(2) transformations gives rise to a further three gauge particles. [4]

(d) What combination of generators would correspond to QED transformations and hence which of your gauge particles combine under symmetry breaking to become the photon? [2]

B3. The Dirac equation for a particle of mass m may be written as

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

where in the Chiral representation

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

and

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

In this representation the four component wave function may be written as

$$\psi = \begin{pmatrix} u_L \\ u_R \end{pmatrix}$$

where u_L and u_R are two component spinors.

(a) Explicitly determine the matrix

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

[4]

(b) Show that acting on ψ with the two projectors $\frac{1}{2}(1 \pm \gamma^5)$ generates four component spinors ψ_L and ψ_R which only depend on u_L and u_R respectively. [2]

(c) Show that in the massless limit the Dirac equation becomes two separate decoupled equations for ψ_L and ψ_R. [3]

(d) Reintroduce the mass term and show explicitly how it introduces terms that couple the two equations for ψ_L and ψ_R [2]

(e) Explain why in the electroweak theory we can not write such a mass term for the down quark. How is the observed down quark mass then produced in that theory? [4]

(f) Since the down quark has the same quantum numbers as the strange quark the most generally gauge invariant form for one of the Dirac equations describing them at low energies takes the form

$$i\partial_\mu \gamma^\mu \begin{pmatrix} d_L \\ s_L \end{pmatrix} - \begin{pmatrix} m_{dd} & m_{ds} \\ m_{sd} & m_{ss} \end{pmatrix} \begin{pmatrix} d_R \\ s_R \end{pmatrix} = 0$$

Explain how we can find a diagonal form for this equation in terms of mass eigenstates.

Hint: a general matrix M can be diagonalized by acting on it with two unitary matrices V and U as $VMU^\dagger$. [3]

(g) How does this diagonalization process explain the weak decay of the K^+ particle to a positron and an electron neutrino? [2]

B4. (a) What are the benefits and drawbacks of colliding protons on protons at the LHC instead of electrons on positrons? [8]

(b) Why, at the high energies the LHC works at, are proton on proton collisions as useful as proton anti-proton collisions? [2]

(c) Name four mechanisms for the production of the Higgs at the LHC. [4]

(d) What are the main sources of background to the detection of the Higgs at the LHC? [3]

(e) Explain what the main signal for Supersymmetry (SUSY) might be at the LHC. [3]

END OF PAPER