SEMESTER 2 EXAMINATION 2008/09

PHYS6011 PARTICLE PHYSICS

Duration: 120 MINS

*Answer **all** questions in **Section A** and two **and only two** questions in **Section B**.*

*Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.*

*A sheet of Physical Constants will be provided with this exam paper. An outline marking scheme is shown in brackets to the right of each question.*

*Only University approved calculators may be used.*

# Section A

**A1.** Dirac proposed the energy ($E$) momentum ($\mathbf{p}$) relation for a particle of mass $m$

$$E = \alpha.\mathbf{p} + \beta m$$

What conditions must the constants $\alpha$ and $\beta$ satisfy for this equation to describe a relativistic particle for which $E^2 - |\mathbf{p}|^2 = m^2$? [4]

**A2.** The wave equation describing non-interacting photons is just Maxwell's equation in free space. In Lorentz gauge, in which $\partial^\mu A_\mu = 0$, it is

$$\partial_\nu \partial^\nu A^\mu = 0$$

Show that, even though the index $\mu$ on $A^\mu$ runs from 0..3, this equation is consistent with there being just two physical polarizations of the photon. [4]

**A3.** The $\Upsilon(4S)$ particle has a mass of 10.6 GeV/$c^2$ and can be created by colliding an electron and positron (with mass of 0.51 MeV/$c^2$).

a) What energy must the electron and/or positron have in the centre of mass to create the $\Upsilon(4S)$? [1]

b) If the positron is at rest in the laboratory, what energy must the electron now have to create the $\Upsilon(4S)$? [3]

**A4.** Write down the Feynman rule for the coupling between a quark and a gluon being careful to define each quantity. [3]

Briefly describe one piece of experimental evidence for the existence of gluons [1]

**A5.** In the electroweak theory there is a Higgs field which is a complex doublet of the SU(2) gauge symmetry and hence has four real degrees of freedom. Briefly explain why there is only one physical Higgs particle in the spectrum. [4]

## Section B

**B1.** A massless Dirac fermion with momentum $\mathbf{p}$ is described by a two-component spinor, $\psi$, that satisfies the spinor equation

$$H\psi = \sigma.\mathbf{p}\psi$$

where $H$ is the Hamiltonian and $\sigma^i$ are the Pauli matrices.

(a) Show that the commutator between $H$ and the angular momentum operators $\mathbf{L}$ are

$$[H, \mathbf{L}] = -i\sigma \times \mathbf{p}$$

[7]

(b) Show that $\mathbf{L} + \sigma/2$ commutes with $H$. You may use the result

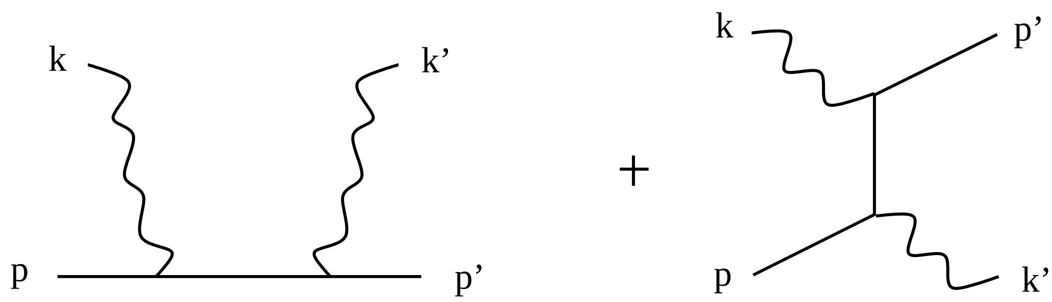
$$[\sigma^i, \sigma^j] = 2i\varepsilon^{ijk}\sigma^k$$

where $\varepsilon^{ijk}$ is a totally anti-symmetric tensor of magnitude one. [7]

(c) What are the physical consequences of the results in parts (a) and (b)? [2]

(d) In the case of a particle moving in the $z$-direction with momentum $p_z$ the eigenvectors of the spinor equation are just $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Find the energy eigenvalues and the eigenvalues of the helicity operator $\sigma.\mathbf{p}/|\mathbf{p}|$. Physically interpret these states. [4]

**B2.** Compton scattering (the scattering of an electron and a photon) is described by the two Feynman diagrams




(a) We normally assume that the external photons in the process satisfy the free Maxwell’s equation

$$\partial^{\nu}\partial_{\nu}A^{\mu} = 0$$

Show that there are solutions of the form $A^{\mu} = \varepsilon^{\mu}e^{-iq^{\lambda}x_{\lambda}}$ with $x^{\lambda}$ the position four-vector and $\varepsilon^{\mu}$ any polarization vector. [2]

(b) Similarly the external electron fields satisfy the free Dirac equation

$$(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0$$

There are solutions of the form $\psi = u(p)e^{-ip^{\lambda}x_{\lambda}}$ - find the equation that $u(p)$ satisfies. [1]

(c) The result at leading order in perturbation theory for the scattering of an electron of charge $q$ from a state $a$ to a state $c$ as the result of the interaction with a U(1) gauge potential $A^\mu$ is

$$\mathcal{M}_{ac} = -i \int J^\mu_{ac}\, A_\mu \, d^4x \qquad \text{where} \qquad J^\mu_{ac} = iq(\bar{\psi}_c \gamma^\mu \psi_a)$$

The internal electron wave function may be found by solving the Dirac equation in the presence of the photon gauge field for $\psi_{\text{int}}$

$$(i\gamma^\mu \partial_\mu - m)\psi_{\text{int}} = q\gamma_\mu A^\mu_{ext} \psi_{ext}$$

where the subscript *ext* corresponds to an external particle at the vertex.

Using these results show that the contribution to the scattering amplitude from the first diagram above is given by

$$\begin{aligned} -i\mathcal{M}_1 \;&=\; \bar{u}(p') \left( \varepsilon^*_\nu(k') \;\; iq\gamma^\nu \;\, \tfrac{i(\not{p}' + \not{k}') + m}{(p'+k')^2 - m^2} \;\, iq\gamma^\mu \;\; \varepsilon_\mu(k) \right) u(p) \\ &\qquad (2\pi)^4 \delta^4(p + k - p' - k') \end{aligned}$$

[10]

(d) Show how this expression can be associated with a set of Feynman rules for the external electron and photon states, the vertices and the internal fermion propagator. [4]

(e) Using your Feynman rules write down the contribution to the scattering amplitude for the second diagram above. [3]

**B3.** (a) How does a Cyclotron work? [8]

(b) What are the limitations of a Cyclotron? [2]

(c) How do the Synchrocyclotron and the Isocyclotron overcome the limitations of the Cyclotron? [2]

(d) What are the benefits and drawbacks of circular and linear colliders? [6]

(e) Name two modern uses (in addition to particle physics research) for Cyclotrons. [2]

TURN OVER

**B4.** A complex scalar field, $\phi$, with mass $m$ and charge $q$ with respect to a U(1) gauge field, $A^\mu$, satisfies the wave equation

$$D^\mu D_\mu \phi + m^2 \phi = 0, \qquad D^\mu = \partial^\mu + iqA^\mu$$

(a) Show that under the gauge transformation

$$\phi \to e^{iq\theta(x)}, \qquad A^\mu \to A^\mu - \partial^\mu \theta(x)$$

$D^\mu \phi$ transforms to $e^{iq\theta(x)} D^\mu \phi$. [6]

(b) Hence, explain why the gauge transformation is a symmetry of the $\phi$ wave equation. [2]

(c) We can introduce a potential for the field $\phi$ of the form

$$V = -\frac{\mu^2}{2}|\phi|^2 + \frac{\lambda}{4}|\phi|^4$$

where $\mu$ and $\lambda$ are positive constants.

Show that this potential is gauge invariant, sketch the potential and find the value of $\phi$ at the minimum. [4]

(d) The gauge field's wave equation in Lorentz gauge is given by

$$\partial^\nu \partial_\nu A^\mu = J^\mu, \qquad J^\mu = iq(\phi^* D^\mu \phi - (D^\mu \phi)^* \phi)$$

Show that if $\phi$ lies at the minimum of the potential in (c) that the photon effectively develops a mass and satisfies the Klein Gordon equation

$$(\partial^\mu \partial_\mu + M^2) A^\mu = 0$$

[3]

(e) Find a solution of this massive Klein Gordon equation for which $A^0 = 0$ and **A** is time independent. Explain, using your solution, why magnetic fields are expelled from the volume of space in which $\phi$ is non-zero and why the theory has zero electrical resistance. [5]

END OF PAPER