SEMESTER 2 EXAMINATION 2007/08

PHYS6011 PARTICLE PHYSICS

Duration: 120 MINS

Answer ***all*** *questions in* ***Section A*** *and two* ***and only two*** *questions in* ***Section B****.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A sheet of Physical Constants will be provided with this exam paper. An outline marking scheme is shown in brackets to the right of each question.

Only University approved calculators may be used.

Section A

A1. At high energies the gauge symmetry of the Standard Model is

$$\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_\mathrm{Y}$$

Explain how each of the left and right handed spinors of the up quark transform under these gauge transformations. [4]

A2. Explain Dirac's interpretation of negative energy states amongst the solutions of the Dirac equation. [4]

A3. What is Synchrotron Radiation? [2]

The radiative energy loss of a particle by Synchrotron Radiation per revolution is given by

$$U_0 = \beta C_\gamma E^4 / R$$

where E is the energy of the particle, R is the radius of the accelerator, and β is the ratio of the particle's speed to the speed of light, v/c. $C_\gamma = 8.85 \times 10^{-5} m/GeV^3$ for electrons and $C_\gamma = 7.78 \times 10^{-18} m/GeV^3$ for protons. Compute the ratio of energy loss per revolution for a 50 GeV electron and a 7 TeV proton in the LEP/LHC accelerator. [2]

A4. Why do Feynman diagrams involving loops of particles give rise to infinities? Briefly explain how renormalization theory can remove these infinities from computations of physical observables. [4]

A5. A gluon is associated with the SU(3) generator

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

in a basis where the quark colour vector is (R, G, B). Draw the Feynman vertex and rules for the two non-zero interactions of this gluon with quarks. [2]

A6. If in some area of space a Higgs field generates a mass, m, for a U(1) gauge field then the gauge field satisfies a massive Klein Gordon equation

$$(\partial^\mu \partial_\mu + m^2) A^\mu = 0$$

Show that static A^μ solutions decay exponentially in that area of space. [2]

Section B

B1. The Dirac equation may be written as

$$i\frac{\partial\psi}{\partial t} = (-i\alpha.\nabla + \beta m)\psi$$

where in the Dirac representation

$$\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(a) Show that $\psi^\dagger\psi$ satisfies an appropriate continuity equation and hence may be interpreted as the probability density. [7]

(b) Show explicitly that

$$(\sigma.\mathbf{p})^2 = |\mathbf{p}|^2$$

where $\mathbf{p}$ is the momentum vector. [4]

(c) Hence show that there are plane wave solutions of the form

$$\psi = \begin{pmatrix} \chi(\mathbf{p}) \\ \phi(\mathbf{p}) \end{pmatrix} e^{-i(Et-\mathbf{p}.\mathbf{x})}$$

with both positive and negative energies. [6]

(d) How does the Feynman Stückelberg interpretation account for the negative energy solutions? [3]

B2. (a) Draw the Feynman diagram for the process

$$e^+e^- \to \mu^+\mu^-$$

and hence, using the Feynman rules, write down an expression for the scattering amplitude for this process. [6]

(b) In order to reconstruct such an e^+e^- collision, the properties of the muons must be measured by a detector. List the component sub-detectors within a typical particle physics detector and explain what they measure. [10]

(c) How would a muon be differentiated from other particles in such a detector? [4]

B3. (a) Describe a piece of experimental evidence for the existence of three colours of quarks. [4]

(b) The existence of three indistguishable colours of each quark implies that the wave function for the three quark colours, $\psi = (R, G, B)$, must be invariant under the action of a 3×3 matrix U,

$$\psi \to U\psi$$

Why must U be unitary? [4]

(c) The free Dirac equation for ψ would be

$$(i\partial^{\mu}\gamma_{\mu} - m)\psi = 0$$

where γ_{μ} are the Dirac gamma matrices and m the fermion's mass. Show the equation is invariant to global transformations of the type above. [2]

(d) U may be parametrized as

$$U \equiv e^{ig_s \theta^a T^a}$$

where θ^a are 8 free parameters, g_s a normalization constant, and the T^a are the generators of the group SU(3).

Show, for infinitessimal θ^a, that to make the Dirac equation for ψ invariant to gauged or local transformations of this type, 8 gluon fields must be introduced. Explicitly show how the gluons will enter the Dirac equation and derive their gauge transformations. [10]

B4. Consider a gauge theory with two U(1) gauge symmetries and a single scalar field ϕ. The scalar field couples to the two gauge bosons with equal but opposite charges. The equations of motion for the two gauge fields are therefore

$$\partial^\nu \partial_\nu A^\mu_{(1)} = iq\phi^* D^\mu \phi - iq(D^\mu\phi)^*\phi$$
$$\partial^\nu \partial_\nu A^\mu_{(2)} = iq\phi^* D^\mu \phi - iq(D^\mu\phi)^*\phi$$

where the covariant derivative is $D^\mu = \partial^\mu + iqA^\mu_{(1)} - iqA^\mu_{(2)}$. The scalar field in addition has a potential

$$V = -\frac{1}{2}\mu^2|\phi|^2 + \frac{1}{4}\lambda|\phi|^4$$

where μ and λ are positive couplings.

(a) Show that the potential is gauge invariant. [2]

(b) Sketch the scalar potential and find the value of ϕ at the potential minimum, v. [4]

(c) Show that the gauge bosons acquire mass and that their mass squared matrix may be written as

$$(A^\mu_{(1)}, A^\mu_{(2)}) 2v^2 \begin{pmatrix} q^2 & -q^2 \\ -q^2 & q^2 \end{pmatrix} \begin{pmatrix} A^\mu_{(1)} \\ A^\mu_{(2)} \end{pmatrix}$$

[6]

(d) Hence show that there is a massless physical gauge field that is an equal superposition of $A^\mu_{(1)}$ and $A^\mu_{(2)}$. [8]

END OF PAPER