SEMESTER 2 EXAMINATION UPDATE EXAMS SCRIPT STYLE

PHYS6011 PARTICLE PHYSICS

Duration: 120 MINS

*Answer **all** questions in **Section A** and two **and only two** questions in **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

Only University approved calculators may be used.

The Pauli matrices are given by

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Section A

A1. Starting from the massless Klein Gordon equation

$$\partial^{\mu}\partial_{\mu}\phi = 0$$

Show that the probability density current

$$J^{\mu} = i(\phi^{*}\partial^{\mu}\phi - \phi\partial^{\mu}\phi^{*})$$

satisfies the conservation equation

$$\partial_{\mu}J^{\mu} = 0$$

[4]

A2. What is the luminosity of a collider with the following parameters: interaction frequency (f): 250x106 per second; number of particles (n): 1010; and bunch size (σ): $5 \times 10^{-6}m$. If the cross-section for producing pairs of τ leptons is 1 nb, how many pairs of τ leptons are produced per second? ($1nb = 10^{-37}m^{2}$) [4]

A3. The Dirac equation for a particle of mass m and charge q coupled to a U(1) gauge potential is

$$(i\gamma^\mu D_\mu - m)\psi = 0, \qquad D_\mu = \partial_\mu + iqA_\mu$$

Show the solutions of this wave equation are invariant under a gauge transformation (parametrized by the function $\theta(x)$):

$$A_\mu \to A_\mu - \partial_\mu \theta(x), \qquad \psi \to e^{iq\theta(x)}\psi$$

[4]

A4. The R ratio is defined in terms of the cross sections, σ, as

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Calculate the value of R between the charm and bottom quark mass energy thresholds. [4]

A5. A real scalar higgs field, ϕ, has the potential

$$V = -m^2\phi^2 + \lambda\phi^4$$

Sketch the potential and find the value of ϕ at the minimum. [2]

Does the model have a Goldstone boson? - explain your answer. [2]

Section B

B1. (a) The Dirac equation can be written as

$$(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0$$

where the γ matrices in the Dirac basis are given by

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

Show that there are static solutions with both positive and negative energy. [6]

(b) How did Dirac interpret the negative energy solutions? [4]

(c) What are the Lorentz transformation properties of $\gamma^{\mu}, \partial^{\mu}$ and m? What does this imply for the wave function ψ? [4]

(d) Show that for the Dirac equation to be invariant under parity ($\mathbf{x} \rightarrow$ - $\mathbf{x}$) anti-particle states must transform to minus themselves. [6]

B2. At leading order in perturbation theory, the amplitude for the scattering of a scalar particle of mass m and charge q from a state a to a state c as the result of the interaction with a U(1) gauge potential A^μ is

$$\mathcal{M}_{ac} = -i \int J^\mu_{ac}\, A_\mu\, d^4x$$

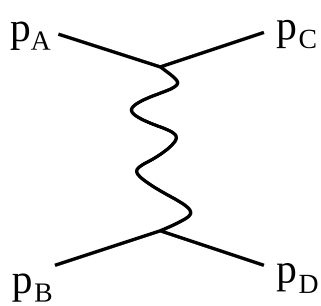
where

$$J^\mu_{ac} = iq(\phi^*_c \partial^\mu \phi_a - \phi_a \partial^\mu \phi^*_c)$$

(a) By solving Maxwell's equation

$$\partial^\nu \partial_\nu A^\mu = J^\mu_{bd}$$

compute the appropriate gauge potential to determine the amplitude for a two to two body scattering of the form



Assume the initial and final state are free solutions of the Klein Gordon equation. Show your result is of the form

$$\mathcal{M}_{abcd} = -i\, q(p_a - p_c)^\mu \left(-\frac{1}{k^2}\right) q(p_b - p_d)_\mu\ \ (2\pi)^4\, \delta^4(p_a + p_b - p_c - p_d)$$

where $k^2 = (p_b - p_d)^2$. [8]

(b) Write down the Feynman rules that would generate your answer. [3]

In the case where the external particles are fermions the amplitude is

$$\mathcal{M}_{abcd} = -i\ \bar{u}_c\, q\gamma^\mu\, u_a \left(-\frac{1}{k^2}\right) \bar{u}_d\, q\gamma_\mu u_d\ \ (2\pi)^4\, \delta^4(p_a + p_b - p_c - p_d)$$

where the u are spinor solutions of the free Dirac equation.

(c) Show using the Dirac equation ($(i\gamma_\mu\partial^\mu - m)\psi = 0$) and the Clifford algebra ($\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$) that

$$\bar{u}_c\gamma^\mu u_a = \frac{1}{2m}\bar{u}_c\left[(p_a + p_c)^\mu + i\sigma^{\mu\nu}(p_c - p_a)_\nu\right] u_a$$

where

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

[7]

(d) Using the result from (c) compare the fermion scattering result to that for the scalar particle scattering. What is the physical meaning of the extra terms? [2]

B3. (a) Explain why QCD has an invariance under the action of unitary 3x3 matrices. [4]

(b) How many degrees of freedom do the 3x3 unitary matrices have? [2]

(c) Explain what the algebra

$$3 \otimes \bar{3} = 8 \oplus 1$$

means when related to a bound state of a quark and an anti-quark. [6]

(d) What is the significance of the product in (c) for the number of physical gluons? [2]

(e) What is a di-quark? [4]

(f) Explain whether it would be possible for there to be a colour neutral particle made of four quarks and one anti-quark. [2]

B4. (a) What is the gauge symmetry of the electroweak interactions and how do the various fermions of the standard model transform under it? Calculate all the appropriate charges. [6]

(b) Explain how a modern particle physics detector is designed to identify the following types of particle:

- electron [3]
- muon [2]
- charged hadron [3]
- neutral hadron [2]
- photon [2]

(c) How might the presence of a neutrino be detected in a particle accelerator detector? [2]

END OF PAPER