UNIVERSITY OF SOUTHAMPTON PHYS6011W1

SEMESTER 2 EXAMINATION 2005/06

PHYS6011 PARTICLE PHYSICS

Duration: 120 MINS

*Answer **all** questions in **Section A** and two **and only two** questions in **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

Only University approved calculators may be used.

The Pauli matrices are given by

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Section A

A1. The angular momentum operator $\mathbf{L}$ does not commute with the Dirac Hamiltonian. The quantity which does commute is

$$\mathbf{J} = \mathbf{L} + \frac{1}{2}\Sigma$$

where Σ can be written in terms of the Pauli matrices σ as

$$\Sigma = \begin{pmatrix} \sigma & 0 \\ 0 & -\sigma \end{pmatrix}$$

What is the significance of this fact and what is the physical interpretation of the eigenvalues of Σ^3? [4]

A2. Define what is meant by a scattering cross section and explain why it provides an experiment independent measure of a particle interaction strength. [4]

A3. An experiment will be constructed utilising an electromagnetic calorimeter made from crystals of BGO (bismuth germanate, density = $7.1 gcm^{-3}$, radiation length = $8.0 gcm^{-2}$). Estimate what thickness (in cm) of BGO is required to completely absorb the energy of an incident 45 GeV electron or photon. (Assume the critical energy in BGO is about 8 MeV) [4]

A4. The 8 gluons of QCD are associated with the generators of SU(3)

$$T^1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T^2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T^3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$T^4 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad T^5 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad T^6 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix},$$

$$T^7 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T^8 = \frac{1}{\sqrt{6}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Show, by considering the Feynman rules for one gluon exchange that a quark and anti-quark of the same colour attract. [4]

A5. What is a virtual particle? [2]

Why do the masses of the W and Z bosons imply the weak force is weak? [2]

Section B

B1. (a) Show that the Klein Gordon equation

$$(\partial^\mu \partial_\mu + m^2)\phi = 0$$

has negative energy solutions. [3]

(b) Dirac proposed the energy momentum relation

$$E = \alpha.\mathbf{p} + \beta m$$

Show that for this to describe a relativistic particle one must have the relations

$$\beta^2 = 1, \qquad \alpha^i \alpha^j + \alpha^j \alpha^i = 2\delta^{ij}, \qquad \alpha^i \beta + \beta \alpha^i = 0$$

[6]

(c) Prove that the α^i and β are traceless [4]

(d) One possible representation of the algebra is

$$\alpha^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Write down the wave equation that Dirac derived from his relation and show that it also has *static* solutions with negative energy. [4]

(e) Describe Dirac's explanation for why a physical particle can not fall into the negative energy states. [3]

B2. (a) Maxwells equations in free space are written in terms of the electromagnetic field tensor

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

Show that $F^{\mu\nu}$ is invariant under gauge transformations

$$A^\mu \to A'^\mu = A^\mu - \partial^\mu \alpha(x)$$

for any function $\alpha(x)$. [2]

(b) Further show that the covariant derivative ($D^\mu = \partial^\mu + iqA^\mu$) acting on a complex, scalar wave function, ϕ, transforms to

$$D^\mu \phi \to e^{iq\alpha(x)} D^\mu \phi$$

when ϕ is rotated in addition by the phase $q\alpha(x)$. [6]

(c) We conclude from (a) and (b) that the Klein Gordon equation coupled to a U(1) electromagnetic potential is gauge invariant. Write down *two* gauge invariant potentials for ϕ, one of which gives a vacuum value of ϕ which is zero, and in the second case where the vacuum value of ϕ is non-zero.

In both cases plot the potential and derive the values of ϕ that minimize the potential. Hence demonstrate that the potentials yield the required vacuum values. [8]

(d) Explain why when the minimum of the potential has a non-zero value of ϕ there is a Goldstone boson. Why does this particle not correspond to a massless state in the spectrum of the theory? [4]

B3. (a) What does it mean that QCD is asymptoptically free? [3]

(b) How does asymptotic freedom help to explain quark confinement? [3]

(c) At LEP with $\sqrt(s) = M_Z$, the cross section $\sigma(e^+e^- \rightarrow Z \rightarrow q\overline{q}) = 30nb$. Since quarks are confined, describe how such an event would physically appear in a detector. [$1bn$ is $10^{-28}m^2$] [2]

(d) The LEP peak luminosity is L = $20 \times 10^{30} cm^{-2} s^{-1}$. Using the cross section information in (c) work out at what rate would such hadronic Z events occur? [2]

(e) Apart from a small correction due to different Z-quark coupling strengths, the branching ratio $Br(Z \rightarrow b\overline{b}) \approx 0.2\ Br(Z \rightarrow q\overline{q})$. Explain why this is the case. [2]

(f) How is it possible for a LEP experiment to distinguish $Z \rightarrow b\overline{b}$ from other flavours of $Z \rightarrow q\overline{q}$? [3]

(g) A theory predicts the existence of a light scalar bottom quark, $\tilde{b}$, which will also be pair produced as $e^+e^- \rightarrow Z \rightarrow \tilde{b}\overline{\tilde{b}}$. This process will be experimentally identical to $e^+e^- \rightarrow Z \rightarrow b\overline{b}$ and so can be measured as a small signal above the large $b\overline{b}$ "background". Estimate the smallest cross section that this process could have and still give a discovery (significance = 5) in 100 pb^{-1} of LEP data. (Assume that the total efficiency of reconstructing and correctly tagging a $b\overline{b}$ or a $\tilde{b}\overline{\tilde{b}}$ event is 50%. Systematic errors may be ignored). [5]

B4. The Standard Model has a SU(2) $\times$ U(1) gauge symmetry group with associated couplings g and g'. This group is broken to the U(1) group of QED by the Higgs mechanism. During this breaking the neutral gauge bosons, $W^{3\mu}$ of the SU(2) group, and B^μ of the original U(1) group acquire masses which can be written as

$$(W^{3\mu}, B^\mu)\,\frac{v^2}{4}\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix}$$

where v is the higgs vacuum expectation value.

(a) Show that if the fields are redefined so that

$$Z^\mu = \cos\theta\, W^{3\mu} - \sin\theta\, B^\mu$$

$$A^\mu = \sin\theta\, W^{3\mu} + \cos\theta\, B^\mu$$

with

$$\sin\theta = \frac{g'}{\sqrt{g^2 + g'^2}}, \qquad \cos\theta = \frac{g}{\sqrt{g^2 + g'^2}}$$

the mass matrix becomes diagonal. [12]

(b) Using the relation for electric charge

$$Q = T_3 + Y/2,$$

where Y is the hypercharge and T_3 the eigenvalue of $\sigma^3/2$, compute the hypercharges of the left and right handed electron. [2]

(c) Show, by considering the Feynman rule for a photon coupling to a right handed electron, that

$$g' \cos\theta = e$$ [3]

(d) Hence show, by considering the Feynman rule for a photon coupling to a left handed electron, that

$$g \sin\theta = e$$ [3]

END OF PAPER