Beyond The Standard Model

Contents

- Neutrino oscillation theory
- Overview of oscillation experiments
- Grand Unified Models
- Hierarchy problems and solutions
- Quantum Gravity and Strings

Learning Outcomes

- Be able to derive the oscillation formulae
- Be able to overview the oscillation experiments
- Section 2+ are not examinable!

Beyond The Standard Model

The Standard Model of particle physics is extremely successful, correctly predicting the results of all experiments so far carried out down to less than a percent accuracy. The model is built around the elegant idea of gauge symmetry and symmetry breaking. Alas little else about the model is elegant! It seems like a work in progress since we have no explanation for

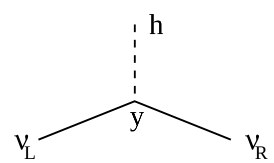
- Why it has the gauge structure $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$.
- Why are the quark and lepton charges what they are.
- Why there are three families of the fermions.
- Why are the fermion masses what they are and so widely different - $m_\nu \sim 10^{-3} eV,\ m_t \simeq 175 GeV$
- The form of higgs potential (if the higgs exists)

In addition we know that eventually we will have to include gravity into our model. In this section we will very briefly look at some of the unproven ideas that have been suggested might play a role in answering these questions.

Before we become completely speculative though there have been a number of recent experiments confirming that neutrinos have mass and therefore that right handed neutrinos exist. Strictly this is "beyond" the original Standard Model.

1 Neutrino Mass

We will see shortly the experiments that confirm neutrinos have mass. To include these masses in the Standard Model is not a challenge - one allows right handed neutrinos which couple to the left handed neutrinos with a higgs. When the higgs gets a vev the mass results.



$m_\nu = y\, v$

1.1 Oscillations

The neutrino masses are extremely small and have not been detected by direct observation. Instead the neutrino species have been observed to oscillate into each other as they propagate. This is in fact a natural consequence of their having mass as we can easily show.

The story is similar to that for the down type quarks. There can be mass terms between different weak eigenstates

$$\nu_L^e \;\text{——}\!\times\!\text{——}\; \nu_R^e \qquad\qquad \nu_L^e \;\text{——}\!\times\!\text{——}\; \nu_R^\mu$$

giving a mass matrix of the form

$$(\nu_e, \nu_\mu)_L \begin{pmatrix} m_{ee} & m_{e\mu} \\ m_{\mu e} & m_{\mu\mu} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}_R \tag{1}$$

We can diagonalize the mass matrix to find the mass eigenstates

$$\begin{aligned} \nu_e^{\text{weak}} &= \cos\theta\ \nu_e^{\text{mass}} + \sin\theta\ \nu_\mu^{\text{mass}} \\ \nu_\mu^{\text{weak}} &= -\sin\theta\ \nu_e^{\text{mass}} + \cos\theta\ \nu_\mu^{\text{mass}} \end{aligned} \tag{2}$$

or

$$\begin{aligned} \nu_e^{\text{mass}} &= \cos\theta\ \nu_e^{\text{weak}} - \sin\theta\ \nu_\mu^{\text{weak}} \\ \nu_\mu^{\text{mass}} &= \sin\theta\ \nu_e^{\text{weak}} + \cos\theta\ \nu_\mu^{\text{weak}} \end{aligned} \tag{3}$$

Now different mass eigenstates propagate differently as we can see from the Dirac equation in free space

$$i\frac{d}{dt}\begin{pmatrix} \nu_e^{\text{mass}} \\ \nu_\mu^{\text{mass}} \end{pmatrix} = H \begin{pmatrix} \nu_e^{\text{mass}} \\ \nu_\mu^{\text{mass}} \end{pmatrix} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \begin{pmatrix} \nu_e^{\text{mass}} \\ \nu_\mu^{\text{mass}} \end{pmatrix} \tag{4}$$

Note that in the mass eigenstate basis the Hamiltonian is diagonal. The solutions are

$$\nu_e^{\text{mass}}(t) = e^{-iE_1 t}\nu_e^{\text{mass}}(0) \tag{5}$$

$$\nu_\mu^{\text{mass}}(t) = e^{-iE_2 t}\nu_\mu^{\text{mass}}(0) \tag{6}$$

Thus if we imagine using the weak force to create a ν_e^{weak} at time $t = 0$ we can see what we're left with later on. In terms of the mass eigenstate basis we've created

$$\cos\theta\ \nu_e^{\text{mass}} + \sin\theta\ \nu_\mu^{\text{mass}}$$

and we know this evolves in time t to

$$\cos\theta\ e^{-iE_1 t}\ \nu_e^{\text{mass}} + \sin\theta\ e^{-iE_2 t}\ \nu_\mu^{\text{mass}}$$

Rewriting back in terms of the weak eigenstates gives now four terms. The two involving the ν_μ^{weak} are

$$-\sin\theta\ \cos\theta\ e^{-iE_1 t}\ \nu_\mu^{\text{weak}} + \cos\theta\ \sin\theta\ e^{-iE_2 t}\ \nu_\mu^{\text{weak}}$$

So there is a non-zero amplitude for finding a muon neutrino at time t. To get the probability we square

$$
\begin{aligned}
P_{e\to\mu}(t) &= \cos^2\theta\ \sin^2\theta \left|e^{-iE_1t} - e^{-iE_2t}\right|^2 \\
&= \sin^2 2\theta\ \sin^2\left[\tfrac{(E_2-E_1)t}{2}\right]
\end{aligned}
\tag{7}
$$

Now momentum conservation ensures all the end products have the same momentum but the energies are given by

$$
E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p} + ... \tag{8}
$$

since the mass is small. We have

$$
\frac{E_2 - E_1}{2} = \frac{m_2^2 - m_1^2}{4E} \tag{9}
$$

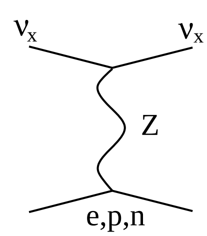
and if we assume that they travel at basically the speed of light so $v = c = 1$ and $t = L$, the distance travelled,

$$
P_{e\to\mu}(t) = \sin^2 2\theta\ \sin^2\left(\frac{\Delta m^2}{4E}L\right) \tag{10}
$$

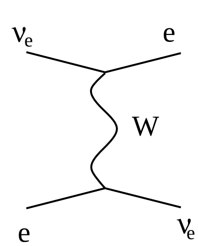
This is the formulae that underlies all oscillation experiments. Note one can only ever hope to measure mass differences not absolute masses from this formulae.

1.1.1 Matter Effects

When neutrinos pass through matter such as in the Sun or Earth there are interactions that effectively adjust their mass. These are not the same though for all neutrino species because matter is made of first generation particles only. Weak interactions via exchange of the Z boson are the same for all families



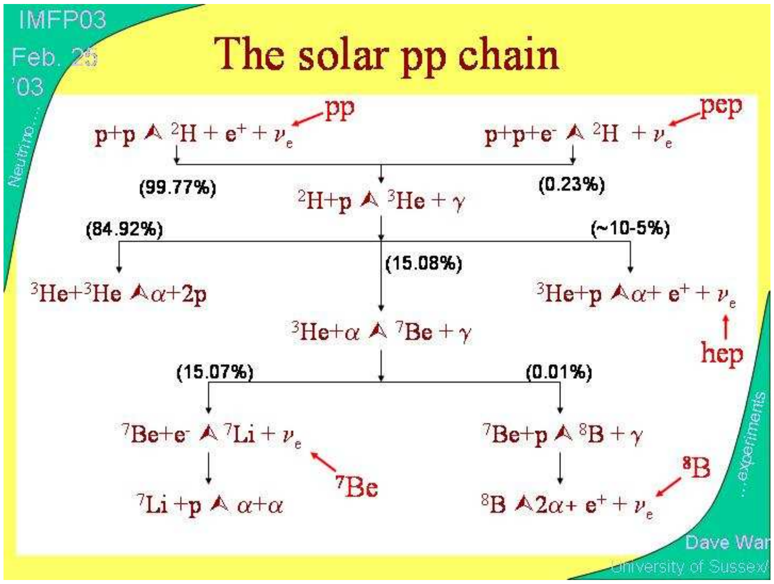
but W exchange diagrams only occur for electron neutrinos



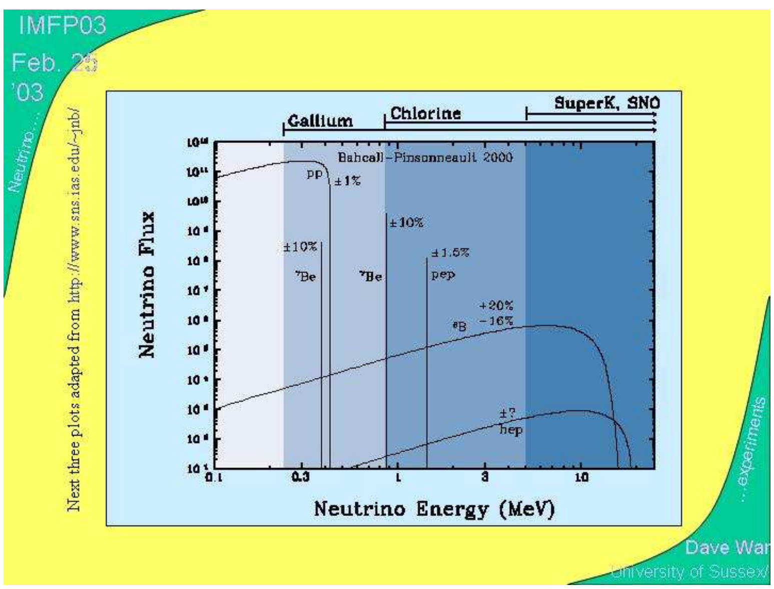
It turns out that for the experiments that have detected neutrino mass these terms are crucial - the oscillations are observed in the matter of the Sun and Earth. We won't go into any more detail here though.

1.2 Solar Neutrino Oscillations - $\nu_e \to \nu_\mu$

The nuclear reactions in the sun produce neutrinos at many different energies from different processes in the solar pp chain. Here are the main reactions



and the predicted flux as a function of energy. We expect a flux at the earth of $6.65 \times 10^{10}/cm^2/s$!



The key experiments to search for this flux are

Chlorine (Ray Davis): a tank of cleaning fluid contains chlorine atoms that when struck by an electron neutrino become Argon

$$\nu_e + Cl_{17}^{37} \rightarrow Ar_{18}^{37} + e^- \tag{11}$$

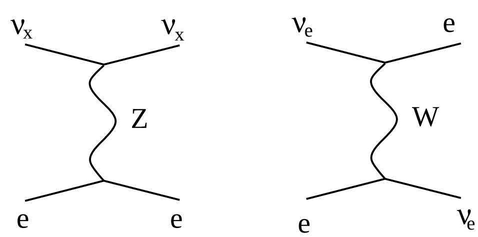
Counting the Argon atoms is hard! The experiment saw a deficit of neutrinos relative to prediction.

SAGE and GALLEX: A block of Gallium slowly converts to Germanium

$$Ga^{71} + \nu_e \rightarrow Ge^{71} + e^- \tag{12}$$

The process then reverses by electron capture giving off a recognizable photon signature. Again a deficit is reported.

Kamioke and Super-Kamiokande: These experiments are just water tanks deep underground. Neutrinos of all types can scatter off electrons by Z boson exchange - the scattered electron gives off Cherenkov light that is detected. Electron neutrinos can also scatter off electrons by W boson exchange



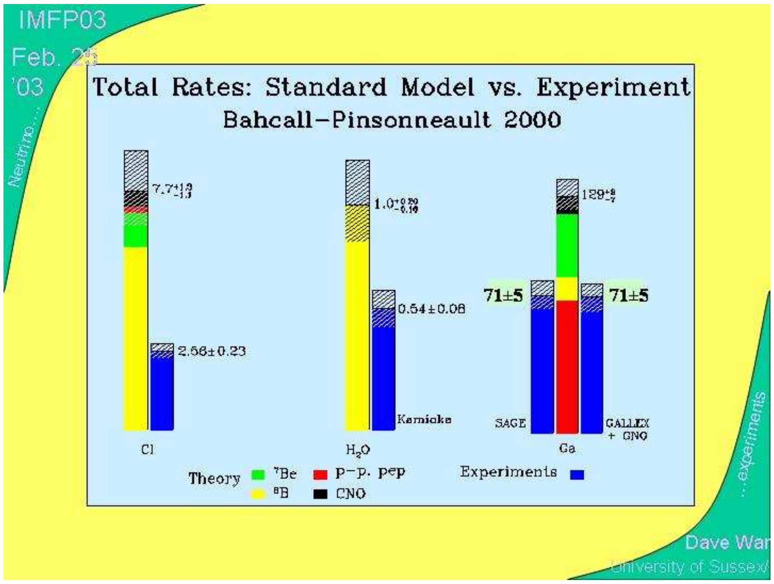
Again the total rate is below that predicted.

SNO: next a tank of D_2O was used. The signals from Super-K are all present but in addition we have now added neutrons that can interact with electron neutrinos

$$\nu_e + d \rightarrow p + p + e^- \tag{13}$$

Combining the deficit numbers of these last two experiments demonstrates that the deficit is in electron type neutrinos whilst the total neutrino number is correct! A clear signal of oscillation.

The data from these experiments is tabulated here



The upshot is

$$\Delta m^2_{\rm sol} \sim 7 \times 10^{-5} eV, \qquad \theta_{\rm sol} \sim 35^o \tag{14}$$

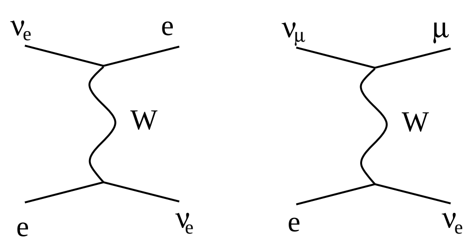
1.3 Atmospheric Neutrino Oscillations - $\nu_\mu \to \nu_\tau$

Cosmic rays hitting the top of the Earth's atmosphere produce pions that then decay to give neutrinos

$$\pi^- \to (\mu^- \to e^- + \nu_\mu + \bar{\nu}_e) + \bar{\nu}_e \tag{15}$$

So we expect to see twice as many ν_μ as ν_e.

The Super-K water tank can tell the difference between electrons and muons in the final state (the Cherenkov light ring is fuzzy for an electron but sharp for a muon) so can distinguish



For neutrinos coming from the sky above the detector the correct ratio of neutrino species is observed. However, for those passing up through the Earth from below the number of the two species is about the same. We believe the muon neutrinos are oscillating into tau neutrinos. The data gives

$$\Delta m^2_{\rm atmos} \sim 3 \times 10^{-3} eV^2, \qquad \theta_{\rm atmos} \sim 45^o \tag{16}$$

2 Grand Unification (GUTs)

The gauge symmetry principle is the over riding message the Standard Model has for us. We have also learnt from the weak force that a gauge symmetry can be hidden from us by the higgs mechanism. It seems natural therefore to try to gauge every possible symmetry in sight! Such theories also try to unify $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ into a single large group so there is only one force. Here are some examples of the things you could try.

2.1 Parity Restoration

The full global symmetry group acting on the doublets

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \qquad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_R \tag{17}$$

is

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{\text{lepton}} \tag{18}$$

here $\mathrm{U}(1)_{\text{lepton}}$ is associated with the generator diag(1,1) which counts lepton number.

We could gauge this full group and restore left right symmetry - there would be 7 gauge bosons. We would then need a higgs that is charged under $SU(2)_R \otimes U(1)_{\text{lepton}}$ that would break these groups to $U(1)_Y$ leaving the Standard Model and 3 heavy gauge bosons. These heavy bosons would mediate right handed β decay suppressed by $1/M_{W'}^2$. As yet there is no evidence for this process.

2.2 SU(4) Colour

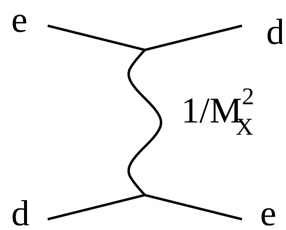
Within a family there are

$$\begin{pmatrix} u \\ d \end{pmatrix}^R, \quad \begin{pmatrix} u \\ d \end{pmatrix}^G, \quad \begin{pmatrix} u \\ d \end{pmatrix}^B, \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix} \tag{19}$$

where the three colours of quarks are written separately. We could make the lepton doublet the fourth colour of an SU(4) group (with 15 gauge bosons). A higgs in the 4 dimensional representation of SU(4) with a vev $(v, 0, 0, 0)$ would break to SU(3).

To do this one would need the quark and lepton doublets to have the same hypercharge. In fact though you can construct hypercharge after the breaking out of the diagonal generator of SU(4) $T^{15} = \mathrm{diag}(1, 1, 1, -3)$ and the generator diag(1,1,1,1). So it can be made to work.

We would predict massive gauge bosons mediating interactions like

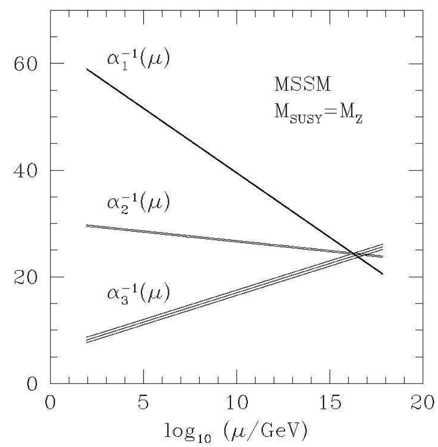


There's no evidence as yet for this.

2.3 SU(5) GUT

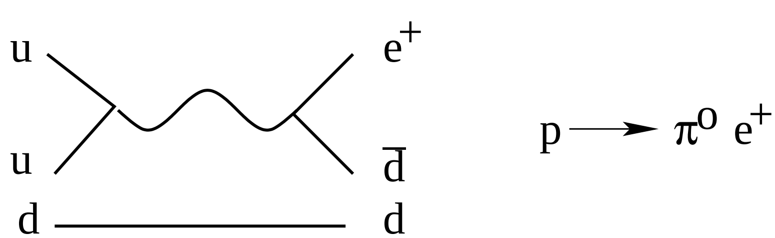
An obvious question to ask is what is the smallest group one can pack $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ into. The answer is SU(5).

If at a high scale there is only one group then there can be only one gauge coupling constant. If we run the gauge couplings of the standard model to very high scales they do in fact converge!



With just the Standard Model particles the couplings in fact miss by 10% or so. One can always include extra particles though to change the running or unify in steps. The diagram above has supersymmetry (which we'll discuss below) included to make things work.

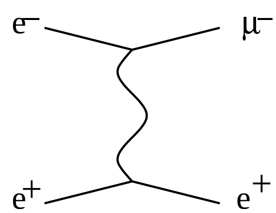
The SU(5) theory allows protons to decay



The Super-K water tank can look for events where protons in the water decay but as yet have no signal. This means the extra SU(5) gauge bosons must have a mass in excess of 10^{16}GeV.

2.4 SU(3) Family Symmetry

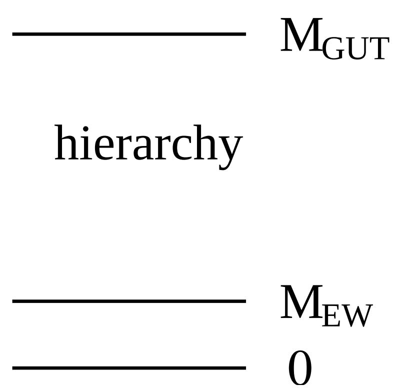
One might wonder if the three families of particles are secretly the charges of an SU(3) gauge group. Here we would not expect the family gauge eigenstates to match the mass eigenstates and one would get Flavour Changing Neutral Currents



These don't exist in the Standard Model and have never been observed. We conclude the massive gauge bosons must be in excess of about 10^5 GeV.

3 Hierarchy Problems

It seems very likely that the full theory of nature includes very high "GUT" energy scales in it. We might wonder why it is that if there are these characteristic energy scales there are particles that are lighter than these scales.

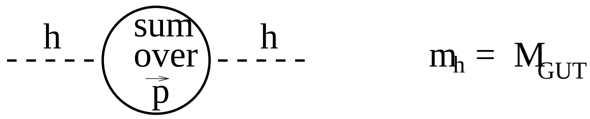


How do we explain the hierarchy between the electroweak scale and the GUT scale?

Gauge Bosons: gauge symmetry enforces the gauge bosons to be massless so a hierarchy is possible.

Fermions: in the absence of mass the left and right handed chiral fermions separate and there is an $SU(N)_L \otimes SU(N)_R$ global symmetry acting on N fermions. If a mass formed it would break this symmetry to a single $SU(N)$. Thus without a symmetry breaking mechanism chiral symmetry enforces masslessness and can maintain a hierarchy.

Scalars: have no symmetry protecting their mass. One could argue this is why we have never seen a fundamental scalar to date. In the Standard Model this shows up for the higgs at loop level where contributions to its mass blow up



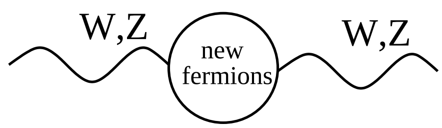
We have to imagine that all the loop contributions and the bare mass cancel to 30 decimal places to get a higgs mass of 1 TeV! Many people argue this is a fundamental flaw with the higgs mechanism.

3.1 Proposed Solutions

Composite Higgs

The pion is a light scalar - it is light because it is a bound state of fermions so above the scale Λ_{QCD} the fermion chiral symmetries act to keep the sector light.

So could the higgs be a bound state of some new fermions, bound by some new force ("techni-colour")? There are such theories and in them we can look for the new fermions in loop diagrams such as

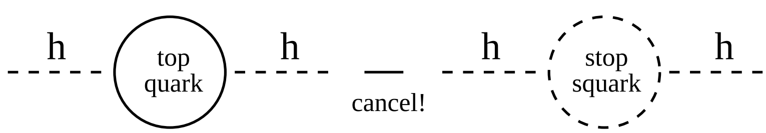


They actively don't seem to be there in the data.

Supersymmetry

Supersymmetry is a new symmetry that links fermions and bosons. For every fermion we must introduce a scalar particle and for every gauge boson a fermion. This doubles the particle spectrum and turns out to introduce about 100 new parameters! Since we haven't seen these partners yet they must be heavy.

The gain from this idea is that the supersymmetry means that the scalar masses are protected from getting large by their fermion partners chiral symmetry. At loop level for the higgs this means loop corrections vanish



This cancellation though will only set in at the mass scale of the super-partners. So if we want the higgs to have a mass of 1 TeV the super-partners must have a mass around there too. If this is all correct we should see supersymmetry at the LHC.

The TOE Is Now!

Another possibility is that there simply is no hierarchy! If the theory of everything is at the electroweak scale then there's nothing to discuss. This seems a bit too good to be true. In any case we don't have any real examples of this type because we can't explain gravity. It would certainly make LHC exciting!

4 Gravity

Lets finally have a brief look at gravity. There is no theory of quantum of gravity that is in anyway tested but interesting ideas have emerged from thinking about what such a theory might look like.

4.1 Newtonian Gravity

Let's begin back at Newtonian gravity where the force is described by a potential

$$\vec{F} = -m\vec{\nabla}\phi, \qquad \phi = -\frac{GM}{r} \tag{20}$$

The strength of the interaction is determined by

$$G = 6.673 \times 10^{11} m^3 kg^{-1} s^{-2} \tag{21}$$

We can construct a dimensionless coupling

$$\alpha_G = \frac{GM^2}{\hbar c} \tag{22}$$

For α_G to be one we require

$$M \simeq 10^{19} GeV/c^2 \tag{23}$$

This is the so called "Planck energy scale". It is a measure of the energy two particles must have for their gravitational interaction to be important in a collider experiment.

4.2 General Relativity

Newtonian gravity has been superseded by General Relativity. In GR the force results from a particle travelling by the shortest distance between points in a curved space.

In the absence of gravity space is flat and described by the metric

$$g_{\mu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{24}$$

or the line element

$$ds^2 = -c^2 dt^2 + dx^2 = dy^2 + dz^2 \tag{25}$$

The action for the particle is the length of its world line

$$\begin{aligned} S &= m \int d\tau \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} \\ &= m \int d\tau \left(c^2 \left(\frac{dt}{d\tau} \right)^2 - \left(\frac{dx^i}{d\tau} \right)^2 \right)^{1/2} \end{aligned} \tag{26}$$

In the non-relativistic limit where $\tau \simeq t$ and speeds are small we have

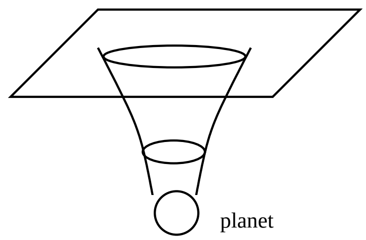
$$S = m \int dtc \left(1 - \frac{1}{2}\frac{\dot{x}^2}{c^2} + ...\right) \tag{27}$$

The Euler Lagrange equation is

$$\frac{d}{dt}\left(\frac{m\dot{x}}{c}\right) = 0 \tag{28}$$

or the familiar

$$m\ddot{x} = 0 \tag{29}$$



When a planet is present to provide a gravitational attraction the space gets deformed so that the line element is

$$ds^2 = -(1+\phi)c^2dt^2 + (1-\phi)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{30}$$

in spherical polar coordinates. Here

$$\phi = \frac{2MG}{c^2r} \tag{31}$$

This returns to flat space far from the planet near $r = 0$ but is curved close to the origin.

The action for a particle is again its world line which here gives

$$S = m \int d\tau \sqrt{(1+\phi)c^2\left(\frac{dt}{d\tau}\right)^2 - (1-\phi)^{-1}\left(\frac{dx^i}{d\tau}\right)^2} \tag{32}$$

To see that this indeed is equivalent to Newtonian Gravity we take the non-relativistic limit $\dot{x} \ll c$ and $\phi \ll 1$

$$S = m \int dtc \left(1 + \frac{1}{2}\phi(x) - \frac{1}{2}\frac{\dot{x}^2}{c^2} + ...\right) \tag{33}$$

The Euler Lagrange equation is

$$-\frac{d}{dt}\left(\frac{m\dot{x}}{c}\right) - cm\frac{d}{dx}\frac{1}{2}\phi = 0 \tag{34}$$

or

$$m\ddot{x} = m\frac{d}{dx}\left(\frac{GM}{r}\right) \tag{35}$$

as we require.

4.3 Gravity Waves

The form of the metric results from Einstein's equations which we will not review here. However it is possible to look for solutions that have small wave-like perturbations in a given background. We set

$$g^{\mu\nu} = g_0^{\mu\nu} + h^{\mu\nu} \tag{36}$$

The resulting linearized equation for $h^{\mu\nu}$ in flat space is

$$\Box h^{\mu\nu} = 0 \tag{37}$$

the Klein Gordon equation for a field that has polarizations which are like the sum of two photon polarizations. If we quantize this theory we expect to get quanta with spin 2 - the graviton.

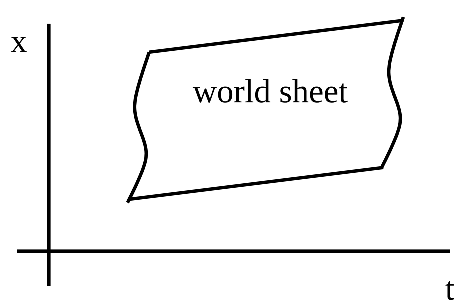
4.4 Quantum Gravity

To get an interacting theory of gravitons we must include interaction terms in addition to the Klein Gordon equation above. The theory that Einstein's equations produces though turns out to be non-renormalizable. At loop level there are more infinities than can be absorbed into the couplings. The answer to everything is infinite!

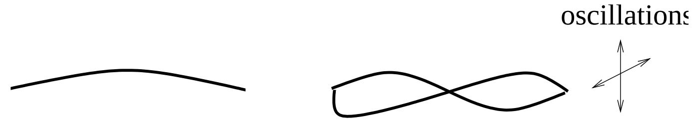
This may mean that Einstein's equations are not the correct description of gravity at short distances... or maybe asking for a quantum theory of gravity is naive because by the time we get to Planck scale distances physics may be very different. This is an open question. Nevertheless it seems interesting to try to develop a consistent quantum theory of gravity - our best attempt so far is string theory.

4.5 String Theory

The fundamental objects in string theory have one dimensional extent and are described by the tension in the string. Probably they are a Planck scale distance in length. The action is just the surface area of the "world sheet" they sweep out in space-time.

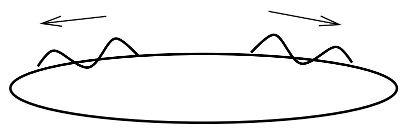


There are two sorts of strings - strings with free ends support wave oscillations of the form



If we look at these strings on long distance scales we see a particle but with two degrees of freedom corresponding to waves in the two transverse directions to the string. The mathematical description turns out to be the same as looking at a gauge boson with two polarizations. These strings look like gauge fields.

We can also have closed loops of string in the theory.



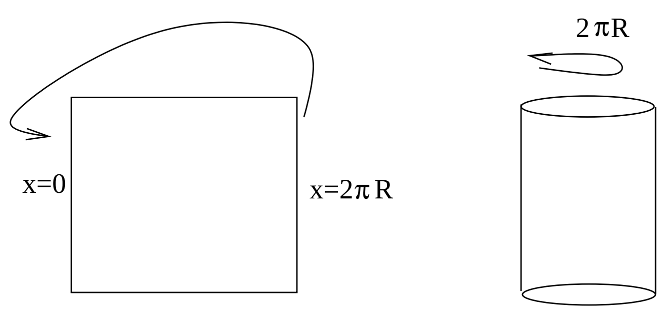
These strings support two independent sets of waves one moving clockwise and one anticlockwise around the string. On long distance scales these look like particles with precisely the polarization structure of the sum of two photons which we need for a graviton! These strings describe gravity.

So string theory allows both gauge and gravity fields which we want. There is little other connection to the Standard Model though we can put the gauge groups we want in by hand.

There is also evidence that this theory is renormalizable. Essentially the Planck length of the string cuts of integrals and stops there being divergences. Actually this only works in 26 dimensional spacetimes, or 10 dimensional spacetimes if supersymmetry is included in the theory.

4.5.1 Compactification

We can remove extra dimensions by making them "circular". We identify the positions x and $x + 2\pi R$



A particles wave function must then satisfy

$$\psi(x) = \psi(x + 2\pi R) \tag{38}$$

so for a free particle

$$e^{ipx} = e^{ip(x+2\pi R)} \tag{39}$$

which tells us that

$$p = \frac{n}{R}, \qquad n = 0, 1, 2... \tag{40}$$

We find a discrete "Kaluza Klein" tower of states with integer spaced masses ($m^2 = p^\mu p_\mu$). Note that as we make R very small the particles moving in the new direction become very massive so the extra dimension becomes invisible.

More recently people have discovered that string theory implies the existence of higher dimensional objects called branes (from membrane). There is a rich structure to this sort of theory. Of course we still have no evidence that any of this is our Universe so let me come to a stop now!