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Semi-Blind Spatial Equalisation for MIMO Channels with Quadrature Amplitude Modulation

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Outline

- Motivations for **semi-blind** detection of **quadrature amplitude modulation** MIMO
- MIMO signal model and proposed semi-blind **spatial equalisation** scheme
- Simulation investigation and performance comparison

Motivations

- Knowledge of **channel state information** is critical to achieve capacity enhancement promised by MIMO, but perfect CSI is often unavailable

- Estimating MIMO channel matrix is a tough job, and **training**-based channel estimation is simple but it reduces achievable throughput

- **Blind** joint channel estimation and data detection does not reduce achievable throughput but is computationally complex

- To resolve **ambiguities** in channel estimation and symbol detection, a few pilot symbols, i.e. some training, are **necessary**

  $\Rightarrow$ Various **semi-blind** joint maximum likelihood (ML) channel estimation and data detection schemes

Motivations (continue)

- Semi-blind iterative least squares channel estimation (LSCE) and ML data detection has attract much attention

  $\Downarrow$ difficult to extend to high-order quadrature amplitude modulation MIMO systems

- Semi-blind **spatial equalisation** offers potentially low-complexity scheme for such MIMO systems

  Existing work (Ding, Ratnarajah & Cowan, 2008, TSP)

- We propose a semi-blind spatial equalisation based on **constant modulus algorithm** assisted **soft decision directed** scheme

  $\Uparrow$ low-complexity **high-performance** $\rightarrow$ approaches **minimum mean square error** solution based on perfect channel state information

Signal Model

- MIMO system of n_T **transmitters**/n_R **receivers**, flat fading channels

$$\mathbf{x}(k) = \mathbf{H}\,\mathbf{s}(k) + \mathbf{n}(k)$$

Transmitted symbol vector $\mathbf{s}(k) = [s_1(k)\ s_2(k) \cdots s_{n_T}(k)]^T$, received signal vector $\mathbf{x}(k) = [x_1(k)\ x_2(k) \cdots x_{n_R}(k)]^T$, channel AWGN vector $\mathbf{n}(k) = [n_1(k)\ n_2(k) \cdots n_{n_R}(k)]^T$, $n_T \leq n_R$

- $n_R \times n_T$ channel matrix $\mathbf{H} = [h_{p,m}]$, $1 \leq p \leq n_R$ and $1 \leq m \leq n_T$

$h_{p,m}$ is a complex Gaussian process with zero mean and $E[|h_{p,m}|^2] = 1$

Block fading, where $h_{p,m}$ is kept constant over small block of N symbols

- M-QAM constellation: $s_m(k) \in \mathcal{S} \stackrel{\triangle}{=} \{s_{i,q} = u_i + j u_q,\ 1 \leq i, q \leq \sqrt{M}\}$ with $\Re[s_{i,q}] = u_i = 2i - \sqrt{M} - 1$ and $\Im[s_{i,q}] = u_q = 2q - \sqrt{M} - 1$

Spatial Equalisation

- Bank of **spatial equalisers** for detecting transmitted symbols $s_m(k)$

$$y_m(k) = \mathbf{w}_m^H \mathbf{x}(k), \ 1 \le m \le n_T$$

- Given **initial training data** $\mathbf{X}_K = [\mathbf{x}(1)\ \mathbf{x}(2) \cdots \mathbf{x}(K)]$ and $\mathbf{S}_K = [\mathbf{s}(1)\ \mathbf{s}(2) \cdots \mathbf{s}(K)]$, **LSCE** of channel $\mathbf{H}$

$$\hat{\mathbf{H}} = [\hat{\mathbf{h}}_1 \cdots \hat{\mathbf{h}}_{n_T}] = \mathbf{X}_K \mathbf{S}_K^H \left(\mathbf{S}_K \mathbf{S}_K^H\right)^{-1}$$

with estimated noise variance $2\hat{\sigma}_n^2 = \frac{1}{K \cdot n_R} \|\mathbf{X}_K - \hat{\mathbf{H}}\mathbf{S}_K\|^2$

- **Initial** spatial equalisers' **weight vectors**

$$\mathbf{w}_m(0) = \left(\hat{\mathbf{H}}\hat{\mathbf{H}}^H + \frac{2\hat{\sigma}_n^2}{\sigma_s^2}\mathbf{I}_{n_R}\right)^{-1} \hat{\mathbf{h}}_m, \ 1 \le m \le n_T$$

- For full rank $\mathbf{S}_K \mathbf{S}_K^H$, $K \ge n_T \Rightarrow$ **minimum training pilots** $K = n_T$

Concurrent Blind Adaptation

- **Concurrent** CMA and SDD equalisers: $\mathbf{w}_m = \mathbf{w}_{m,c} + \mathbf{w}_{m,d}$ with initial $\mathbf{w}_{m,c}(0) = \mathbf{w}_{m,d}(0) = 0.5\mathbf{w}_m(0)$
- **Constant modulus algorithm**:
  - Given spatial equaliser's output $y_m(k) = \mathbf{w}_m^H(k)\mathbf{x}(k)$ at sample k

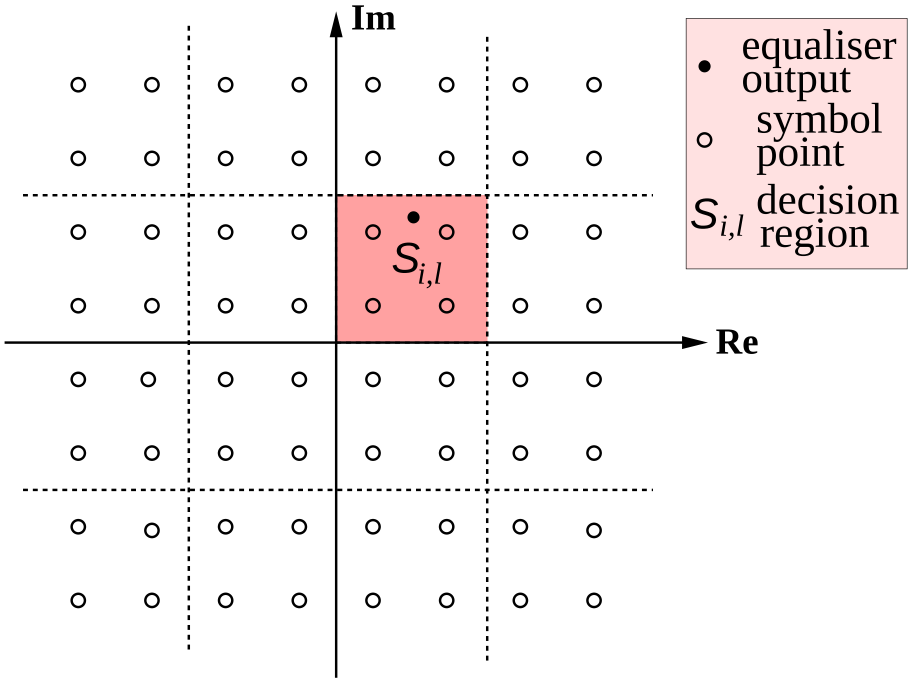
$$\left.\begin{aligned} \varepsilon_m(k) &= y_m(k)\left(\Delta - |y_m(k)|^2\right), \\ \mathbf{w}_{m,c}(k+1) &= \mathbf{w}_{m,c}(k) + \mu_{\text{CMA}}\varepsilon_m^*(k)\mathbf{x}(k), \end{aligned}\right\}$$

  - $\Delta = E\left[|s_i(k)|^4\right] / E\left[|s_i(k)|^2\right]$ and μ_{CMA} is step size
- **Soft decision directed equaliser**: maximise **marginal PDF**

$$J_{\text{LMAP}}(\mathbf{w}_m, y_m(k)) = \rho \log\left(\hat{p}(\mathbf{w}_m, y_m(k))\right)$$

of spatial equaliser's output based on **stochastic gradient** optimisation

Soft Decision Directed Scheme



- Phasor plane is divided into $M/4$ **regions**

$$\mathcal{S}_{i,l} = \{s_{p,q},\ p = 2i-1, 2i,\ q = 2l-1, 2l\}$$

- If $y_m(k) \in \mathcal{S}_{i,l}$, **local approximation** of marginal PDF of $y_m(k)$ is

$$\hat{p}(\mathbf{w}_m, y_m(k)) \approx \sum_{p=2i-1}^{2i} \sum_{q=2l-1}^{2l} \frac{1}{8\pi\rho} e^{-\frac{|y_m(k)-s_{p,q}|^2}{2\rho}}$$

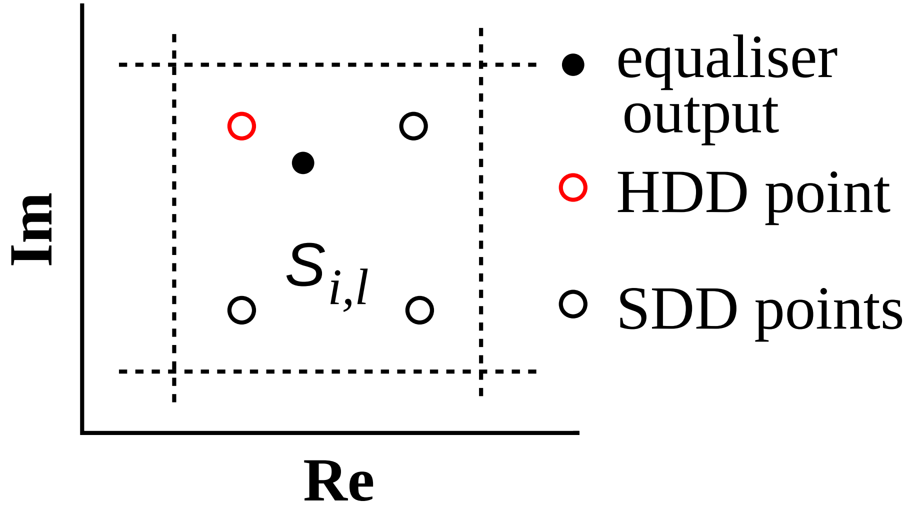
- SDD **weight updating**:

$$\mathbf{w}_{m,d}(k+1) = \mathbf{w}_{m,d}(k) + \mu_{\text{SDD}} \frac{\partial J_{\text{LMAP}}(\mathbf{w}_m(k), y_m(k))}{\partial \mathbf{w}_{m,d}}$$

SDD Scheme (continue)

- μ_{SDD} is **step size** and ρ **cluster width**: when equalisation is done, $y_m(k) \approx s_m(k) + e_m(k)$, where $e_m(k)$ is Gaussian distributed with zero mean and variance $2\sigma_n^2 \mathbf{w}_m^H \mathbf{w}_m$

$$\rho \propto 2\sigma_n^2 \mathbf{w}_m^H \mathbf{w}_m$$



- **Soft** DD nature

$$\frac{\partial J_{\text{LMAP}}(\mathbf{w}_m, y_m(k))}{\partial \mathbf{w}_{m,d}} = \frac{1}{Z_N} \sum_{p=2i-1}^{2i} \sum_{q=2l-1}^{2l} e^{-\frac{|y_m(k)-s_{p,q}|^2}{2\rho}} (s_{p,q} - y_m(k))^* \mathbf{x}(k)$$

with normalisation

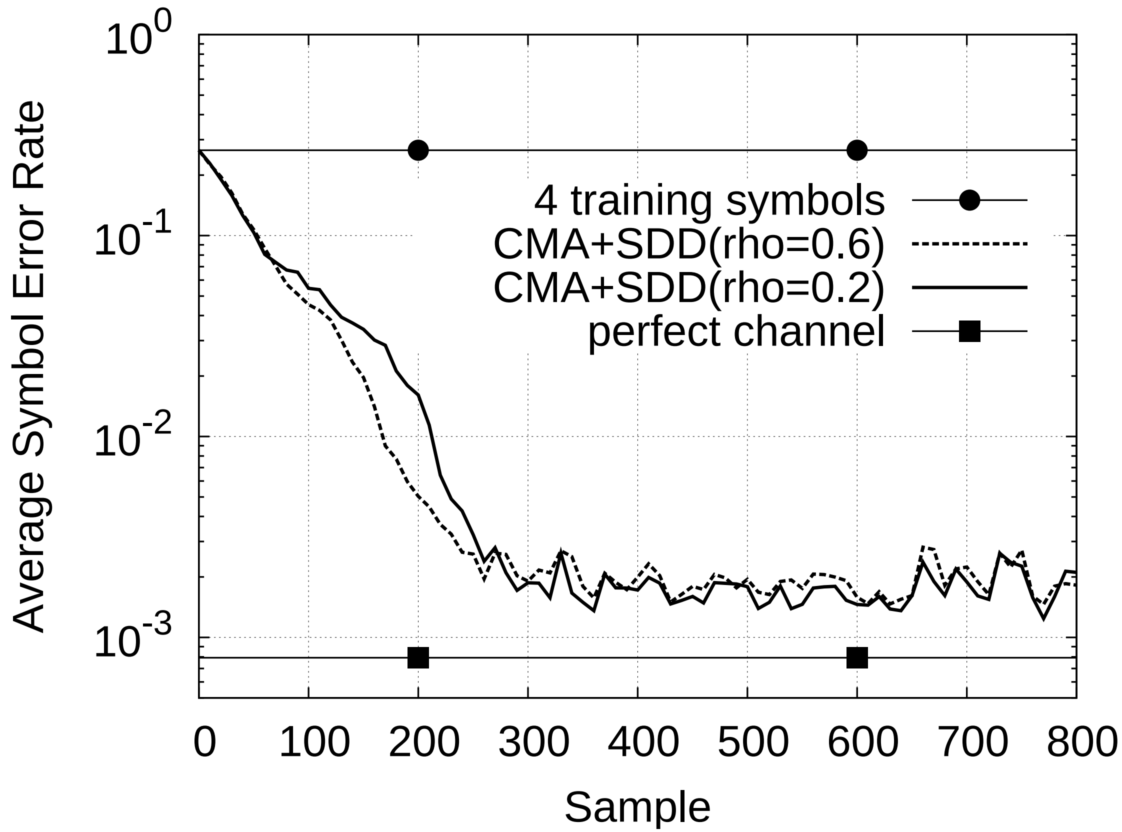
$$Z_N = \sum_{p=2i-1}^{2i} \sum_{q=2l-1}^{2l} e^{-\frac{|y_m(k)-s_{p,q}|^2}{2\rho}}$$

Stationary MIMO Example

- **Stationary** 4×4 MIMO with 64 QAM, training pilots $K = 4$

$-1.4-0.6j$	$0.5+1.1j$	$0.4-0.8j$	$-0.6-0.3j$
$1.7-0.3j$	$1.3-0.3j$	$-0.1-1.4j$	$-0.6-0.5j$
$1.0+0.5j$	$-0.6+0.8j$	$-0.6-0.2j$	$-0.3+0.2j$
$1.2-1.3j$	$-0.7+1.0j$	$0.9-0.3j$	$-0.1+0.7j$

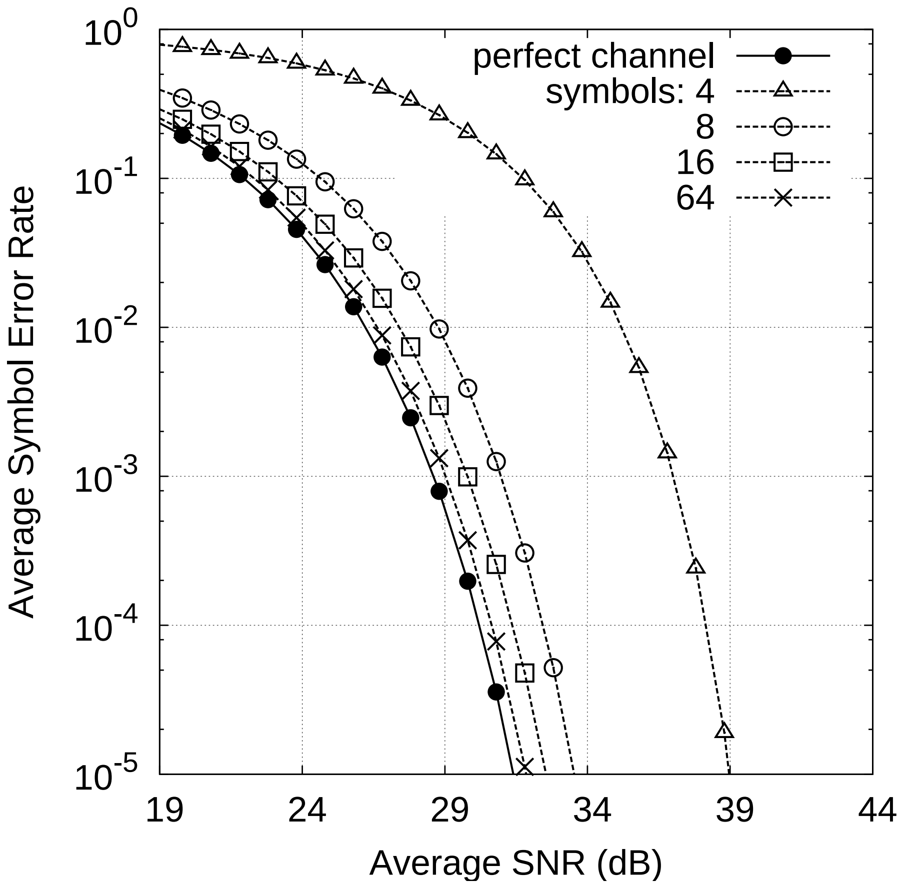
- **Learning curve** of semi-blind CMA+SDD averaged over 10 runs and over all four spatial equalisers: average SNR≈ 29 dB, $\mu_{\text{CMA}} = 4 \times 10^{-7}$, $\mu_{\text{SDD}} = 2 \times 10^{-4}$

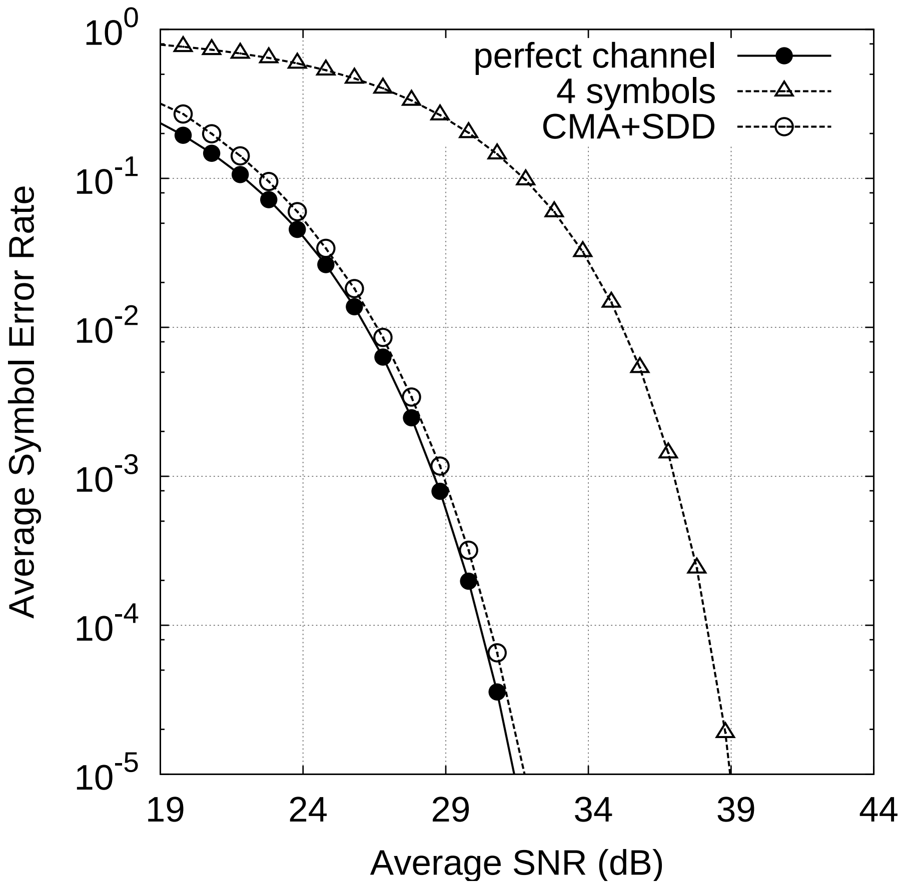


Stationary MIMO Example (continue)

Average **symbol error rates** of spatial equalisation (a) **training-based** given different numbers of training symbols, and (b) **semi-blind** CMA+SDD, in comparison with **minimum mean square error** solution based on perfect channel knowledge

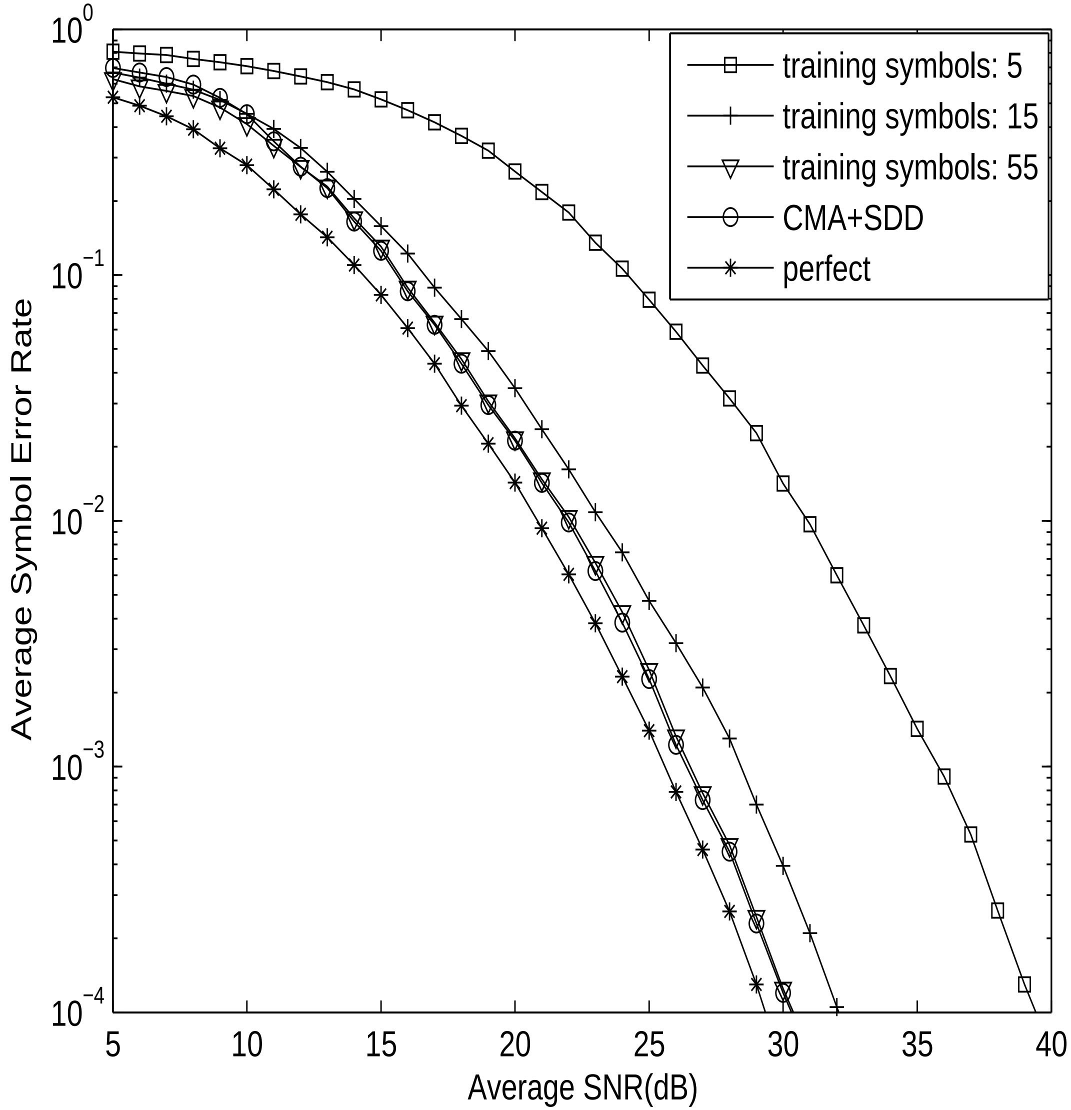
(a) (b)





Block Rayleigh Fading MIMO Example

- 5×4 MIMO with 16-QAM, simulated channel taps $h_{l,m}$, $1 \le l \le 5$ and $1 \le m \le 4$, were i.i.d. complex-valued Gaussian processes with zero mean and $E\left[|h_{l,m}|^2\right] = 1$
- Performance averaged over 100 channel realisations
- Number of pilot symbols $K = 5$, $\mu_{\text{CMA}} = 2 \times 10^{-6}$, $\mu_{\text{SDD}} = 5 \times 10^{-4}$ and $\rho = 0.5$
- Blind adaptive process typically converged within 300 samples



Conclusions

- A low-complexity high-performance semi-blind spatial equalisation scheme has been proposed for high-order QAM MIMO
- Minimum number of pilot symbols, equal to the number of transmit antennas, are used for initial training
- Constant modulus algorithm assisted soft decision directed scheme is apply for blind adaptation
- The scheme converges fast and is capable of approaching the optimal MMSE solution based on perfect channel knowledge
- Effectiveness of proposed semi-blind spatial equalisation scheme has been demonstrated using simulation

THANK YOU.

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