Adaptive Minimum-BER Linear Multiuser Detection

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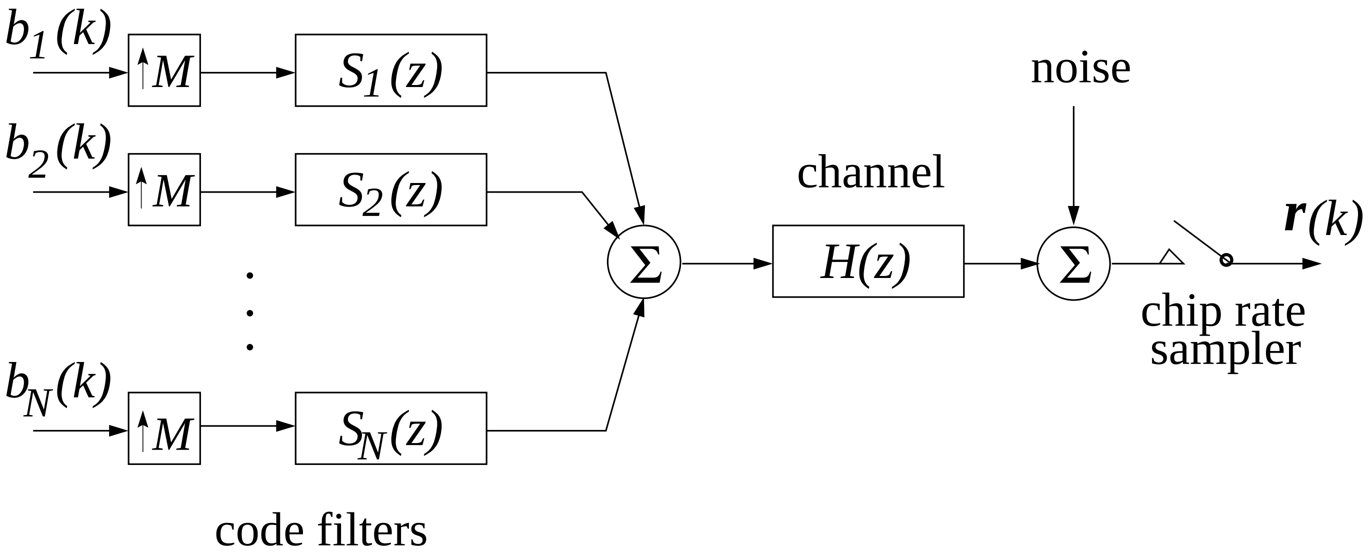
System Model

Downlink synchronous, N-user and M-chip per bit



$$\mathbf{r}(k) = \mathbf{P} \left[\begin{array}{c} \mathbf{b}(k) \\ \mathbf{b}(k-1) \\ \vdots \\ \mathbf{b}(k-L+1) \end{array} \right] + \mathbf{n}(k) = \bar{\mathbf{r}}(k) + \mathbf{n}(k)$$

where the user bit vector $\mathbf{b}(k) = [b_1(k) \ \cdots \ b_N(k)]^T$, L is the ISI span, the Gaussian noise vector $\mathbf{n}(k) = [n_1(k) \cdots n_M(k)]^T$ with zero mean vector and

$$E[\mathbf{n}(k)\mathbf{n}^T(k)] = \sigma_n^2 \mathbf{I} \, ,$$

the $M \times LN$ system matrix

$$\mathbf{P} = \mathbf{H} \begin{bmatrix} \bar{\mathbf{S}}\mathbf{A} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{S}}\mathbf{A} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \bar{\mathbf{S}}\mathbf{A} \end{bmatrix} ,$$

the user unit-length signature sequence matrix $\bar{\mathbf{S}} = [\bar{\mathbf{s}}_1 \ \cdots \ \bar{\mathbf{s}}_N]$, the diagonal user signal amplitude matrix $\mathbf{A} = \mathrm{diag}\{A_1 \ \cdots \ A_N\}$, and the $M \times LM$ CIR matrix $\mathbf{H}$

$$\mathbf{H} = \begin{bmatrix} h_0 & h_1 & \cdots & h_{n_h-1} & & & & \\ & h_0 & h_1 & \cdots & h_{n_h-1} & & & \\ & & \ddots & \ddots & \cdots & \ddots & & \\ & & & h_0 & h_1 & \cdots & h_{n_h-1} & \end{bmatrix}$$

Linear Detector

Linear detector for user i

$$\hat{b}_i(k) = \text{sgn}(y(k)) \text{ with } y(k) = \mathbf{w}^T \mathbf{r}(k)$$

where $\mathbf{w} = [w_1 \ \cdots \ w_M]^T$ is the detector weight vector.

- MMSE solution most widely used, with LMS adaptive implementation.
- There are $N_b = 2^{LN}$ combinations of $[\mathbf{b}^T(k) \ \mathbf{b}^T(k-1) \ \cdots \ \mathbf{b}^T(k-L+1)]^T$:

$$\mathbf{b}^{(j)} = \begin{bmatrix} \mathbf{b}^{(j)}(k) \\ \mathbf{b}^{(j)}(k-1) \\ \vdots \\ \mathbf{b}^{(j)}(k-L+1) \end{bmatrix}, \ 1 \le j \le N_b$$

with $b_i^{(j)}$ the ith element of $\mathbf{b}^{(j)}(k)$.

- $\bar{\mathbf{r}}(k)$ only takes value from the noise-free signal state set:

$$\mathbf{r}_j = \mathbf{P}\mathbf{b}^{(j)}, \ 1 \leq j \leq N_b$$

- The detector $y(k) = y^{'}(k) + n^{'}(k)$, with $= y^{'}(k)$ only takes value from the set:

$$y_j = \mathbf{w}^T \mathbf{r}_j, \ 1 \leq j \leq N_b$$

$n^{'}(k)$ is Gaussian with zero mean and variance $\sigma_n^2 \mathbf{w}^T \mathbf{w}$.

Motivations for Adaptive MBER

- MMSE can be inferior to MBER

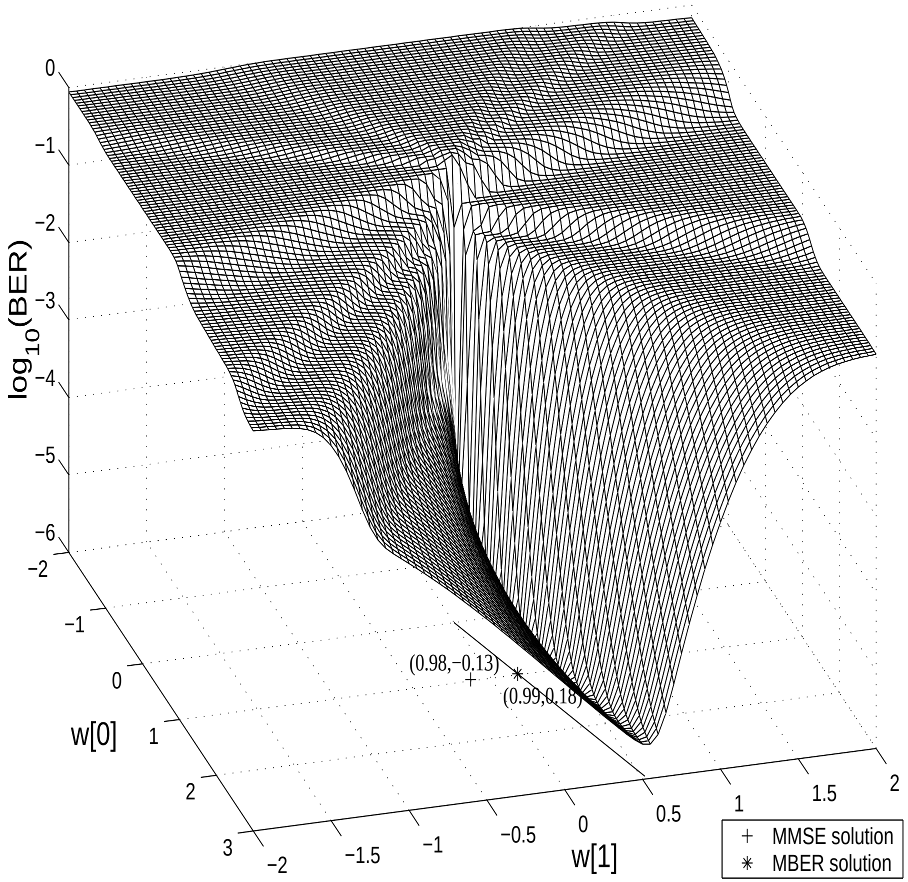
Two equal power users with chip codes $(+1, +1)$ and $(+1, -1)$

Transfer function of CIR
$H(z) = 1 + 0.8z^{-1} + 0.6z^{-2}$

$\text{SNR}_1 = 25$ dB
BER surface for user 1

MMSE solution: $\log_{10}(BER) = -3.88$

MBER solutions: $\log_{10}(BER) = -5.56$



- LMS-style stochastic gradient adaptation

⋆ Two existing stochastic gradient adaptive MBER algorithms

1. Difference approximation MBER, DMBER, (Trans COM **47** (7), pp.1092–1102, 1999)

   Difference approximation for gradient of one-bit error measure, no need for noise pdf assumption, complexity $O(M^2)$, very low convergence rate for small BER

2. Approximate or Adaptive MBER, AMBER, (Globecom'98, pp.3590–3595)

   Like signed-error LMS but modified to continue updating weights in vicinity of decision boundary, very simple with a complexity $O(M)$

⋆ Our approach, LBER, based on kernel density estimation of BER from training data

Also a complexity $O(M)$, simpler than DMBER but more complex than AMBER

Theoretical MBER Solution

Define the signed decision variable

$$y_s(k) = \text{sgn}(b_i(k))y(k) = \text{sgn}(b_i(k))\left(y^{'}(k) + n^{'}(k)\right)$$

with p.d.f.:

$$p_y(y_s) = \frac{1}{N_b\sqrt{2\pi}\sigma_n\sqrt{\mathbf{w}^T\mathbf{w}}}\sum_{j=1}^{N_b}\exp\left(-\frac{(y_s - \text{sgn}(b_i^{(j)})y_j)^2}{2\sigma_n^2\mathbf{w}^T\mathbf{w}}\right)$$

Thus error probability of linear detector:

$$P_E(\mathbf{w}) = \text{Prob}\{\text{sgn}(b_i(k))y(k) < 0\} = \int_{-\infty}^{0} p_y(y_s)\, dy_s = \frac{1}{N_b}\sum_{j=1}^{N_b} Q\left(c_j(\mathbf{w})\right)$$

where

$$Q(y) = \frac{1}{\sqrt{2\pi}} \int_y^\infty \exp\left(-\frac{x^2}{2}\right) dx \quad \text{and} \quad c_j(\mathbf{w}) = \frac{\text{sgn}(b_i^{(j)}) y_j}{\sigma_n \sqrt{\mathbf{w}^T \mathbf{w}}} = \frac{\text{sgn}(b_i^{(j)}) \mathbf{w}^T \mathbf{r}_j}{\sigma_n \sqrt{\mathbf{w}^T \mathbf{w}}}$$

Gradient

$$\nabla P_E(\mathbf{w}) = \frac{1}{N_b \sqrt{2\pi} \sigma_n} \left(\frac{\mathbf{w}\mathbf{w}^T - \mathbf{w}^T \mathbf{w} \mathbf{I}}{(\mathbf{w}^T \mathbf{w})^{\frac{3}{2}}} \right) \sum_{j=1}^{N_b} \exp\left(-\frac{y_j^2}{2\sigma_n^2 \mathbf{w}^T \mathbf{w}} \right) \text{sgn}(b_i^{(j)}) \mathbf{r}_j$$

By normalizing $\mathbf{w}$ to unit length,

$$\nabla P_E(\mathbf{w}) = \frac{1}{N_b \sqrt{2\pi} \sigma_n} \sum_{j=1}^{N_b} \exp\left(-\frac{y_j^2}{2\sigma_n^2} \right) \text{sgn}(b_i^{(j)}) (\mathbf{w} y_j - \mathbf{r}_j)$$

• Steepest-descent or conjugate gradient algorithm ⇒ MBER solution •

Block-data Based Adaptation

Estimate $p_s(y_s)$ based on training data $\{\mathbf{r}(k), b_i(k)\}_{k=1}^{K}$ (kernel density estimation):

$$\hat{p}_y(y_s) = \frac{1}{K\sqrt{2\pi}\rho_n\sqrt{\mathbf{w}^T\mathbf{w}}}\sum_{k=1}^{K}\exp\left(-\frac{(y_s - \text{sgn}(b_i(k))y(k))^2}{2\rho_n^2\mathbf{w}^T\mathbf{w}}\right)$$

where the radius parameter ρ_n is related to the noise standard deviation σ_n

$$\bullet\ \hat{p}_y(y_s) \Rightarrow \hat{P}_E(\mathbf{w}) \Rightarrow \nabla\hat{P}_E(\mathbf{w})\ \bullet$$

$\star$ Gradient algorithm $\Rightarrow$ estimated MBER solution $\star$

Remark: This is analogous to estimated MMSE solution – sample estimates of autocorrelation matrix and cross-correlation vector replacing corresponding ensemble averages

Stochastic Gradient Adaptation

One-sample estimate of p.d.f. and instantaneous stochastic gradient ⇒ LBER

- Re-scaling weight vector (to unit length)

$$\mathbf{w}(k) := \frac{\mathbf{w}(k)}{\sqrt{\mathbf{w}^T(k)\mathbf{w}(k)}}$$

- Detector output

$$y(k) = \mathbf{w}^T(k)\mathbf{r}(k)$$

- Weight update

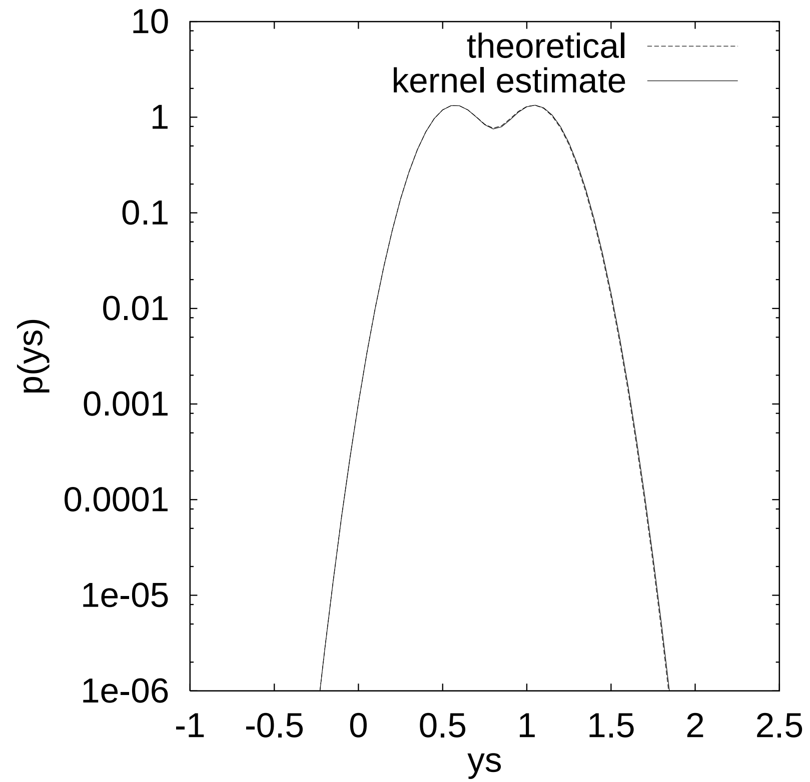
$$\mathbf{w}(k+1) = \mathbf{w}(k) + \frac{\mu}{\sqrt{2\pi}\rho_n} \exp\left(-\frac{y^2(k)}{2\rho_n^2}\right) \mathrm{sgn}(b_i(k))(\mathbf{r}(k) - \mathbf{w}(k)y(k))$$

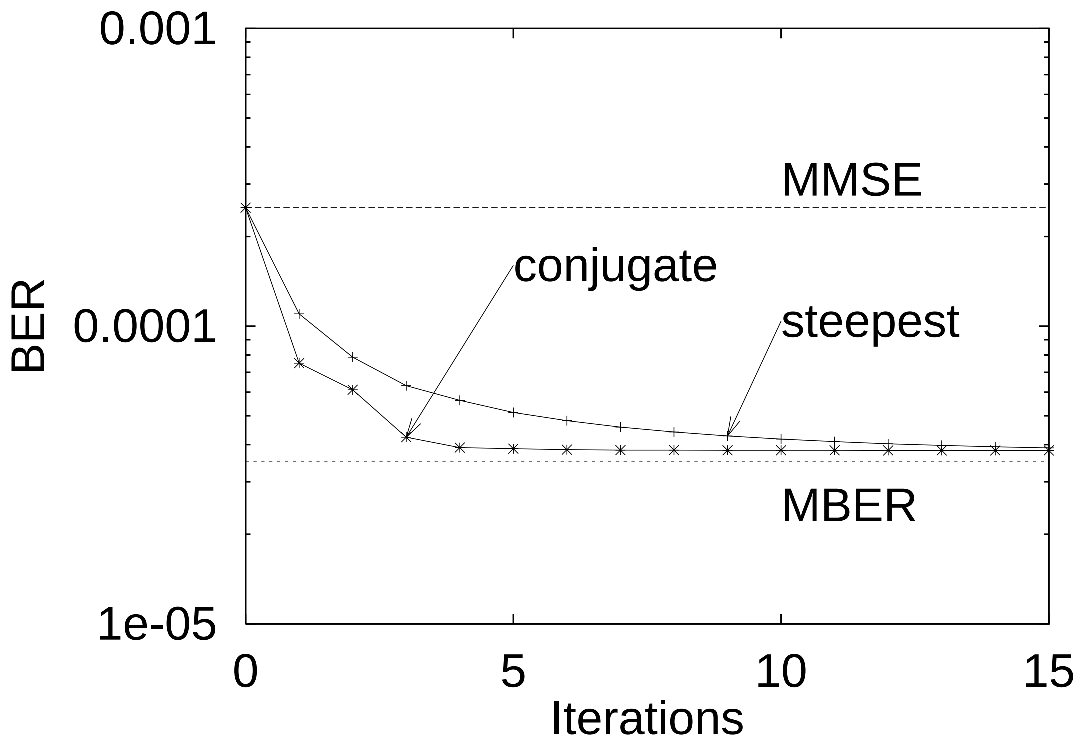
Step size μ and width ρ_n are two algorithm parameters

Simulation

Example 1 Two equal-power users with $(+1, +1, -1, -1)$ and $(+1, -1, -1, +1)$, respectively, and the CIR transfer function $H(z) = 1.0 + 0.25z^{-1} + 0.5z^{-3}$

Data block: 100 samples, $\text{SNR}_1 = \text{SNR}_2 = 16.5$ dB, block adaptation for **user 1**:

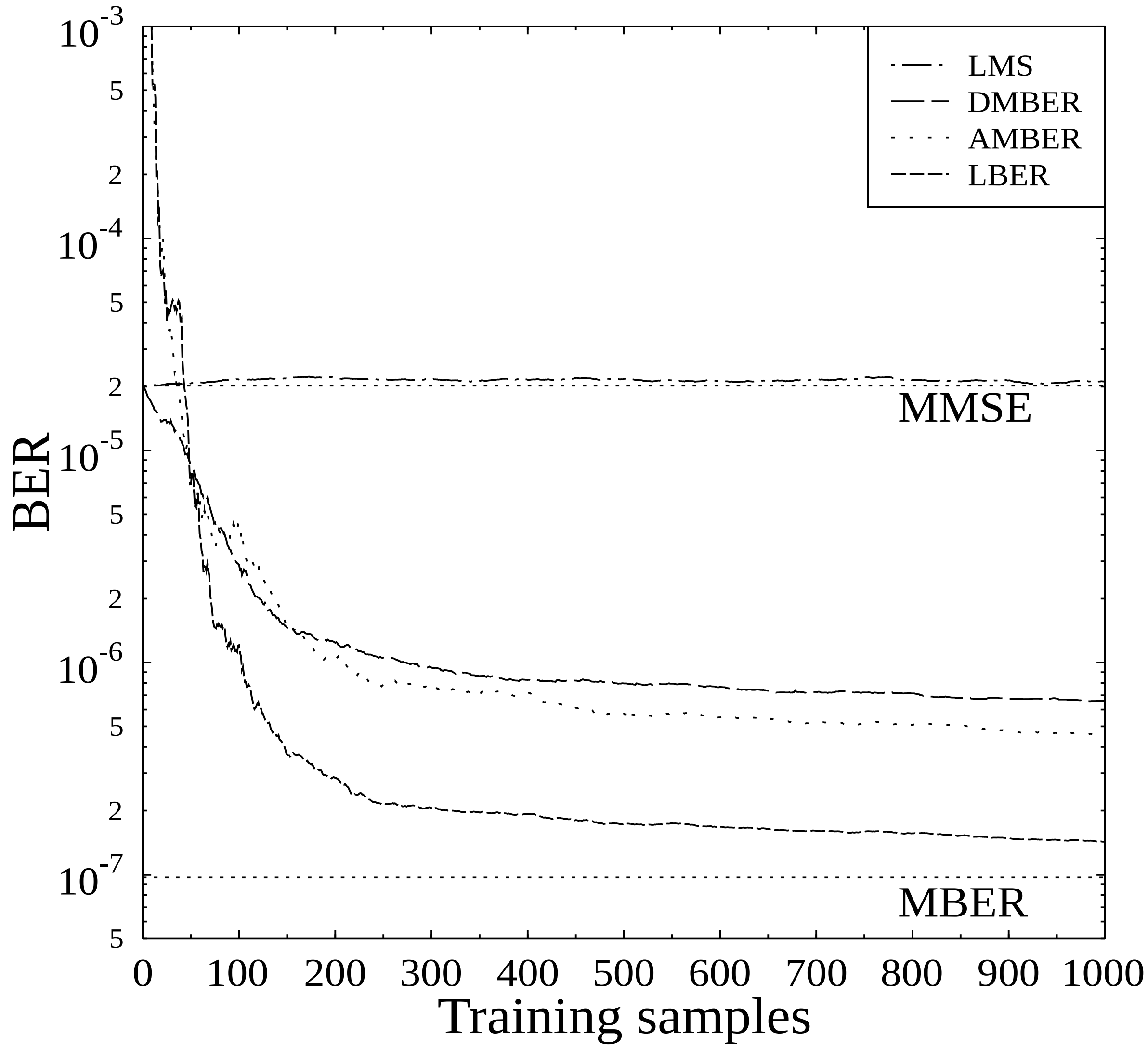




$\text{SNR}_1 = \text{SNR}_2 = 19$ dB

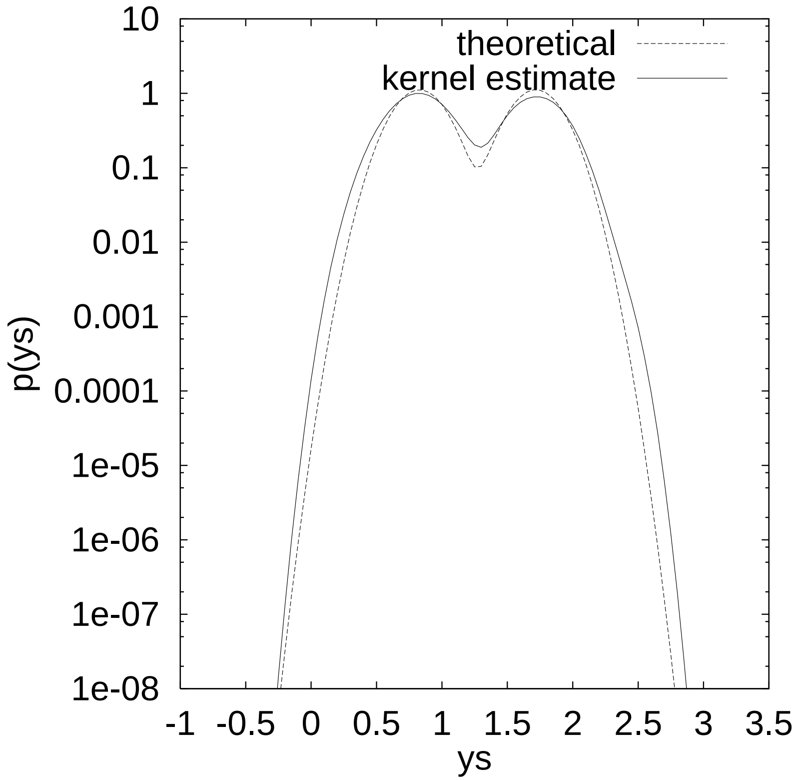
Stochastic gradient adaptation for **user 1**:

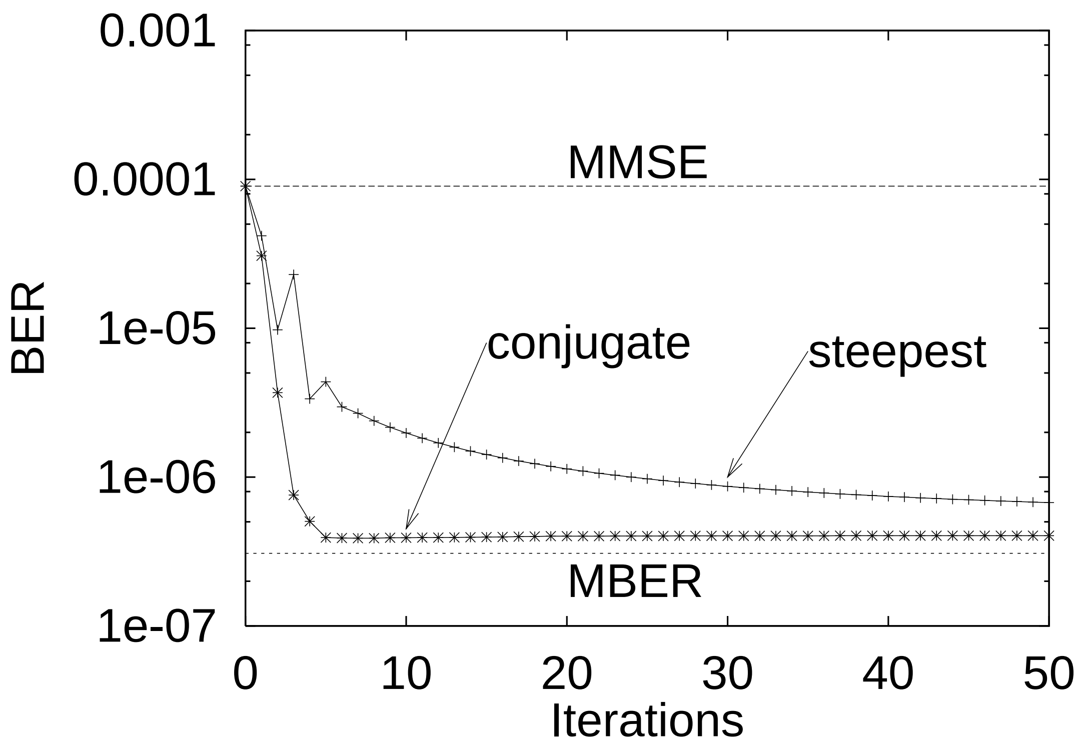
Average over 100 runs



Example 2 Four equal-power users with $(+1, +1, +1, +1, -1, -1, -1, -1)$, $(+1, -1, +1, -1, -1, +1, -1, +1)$, $(+1, +1, -1, -1, -1, -1, +1, +1)$ and $(+1, -1, -1, +1, -1, +1, +1, -1)$; the CIR transfer function $H(z) = 0.4 + 0.7z^{-1} + 0.4z^{-2}$

Data block: 1500 samples, $\text{SNR}_i = 16$ dB for all i, block adaptation for **user 1**:

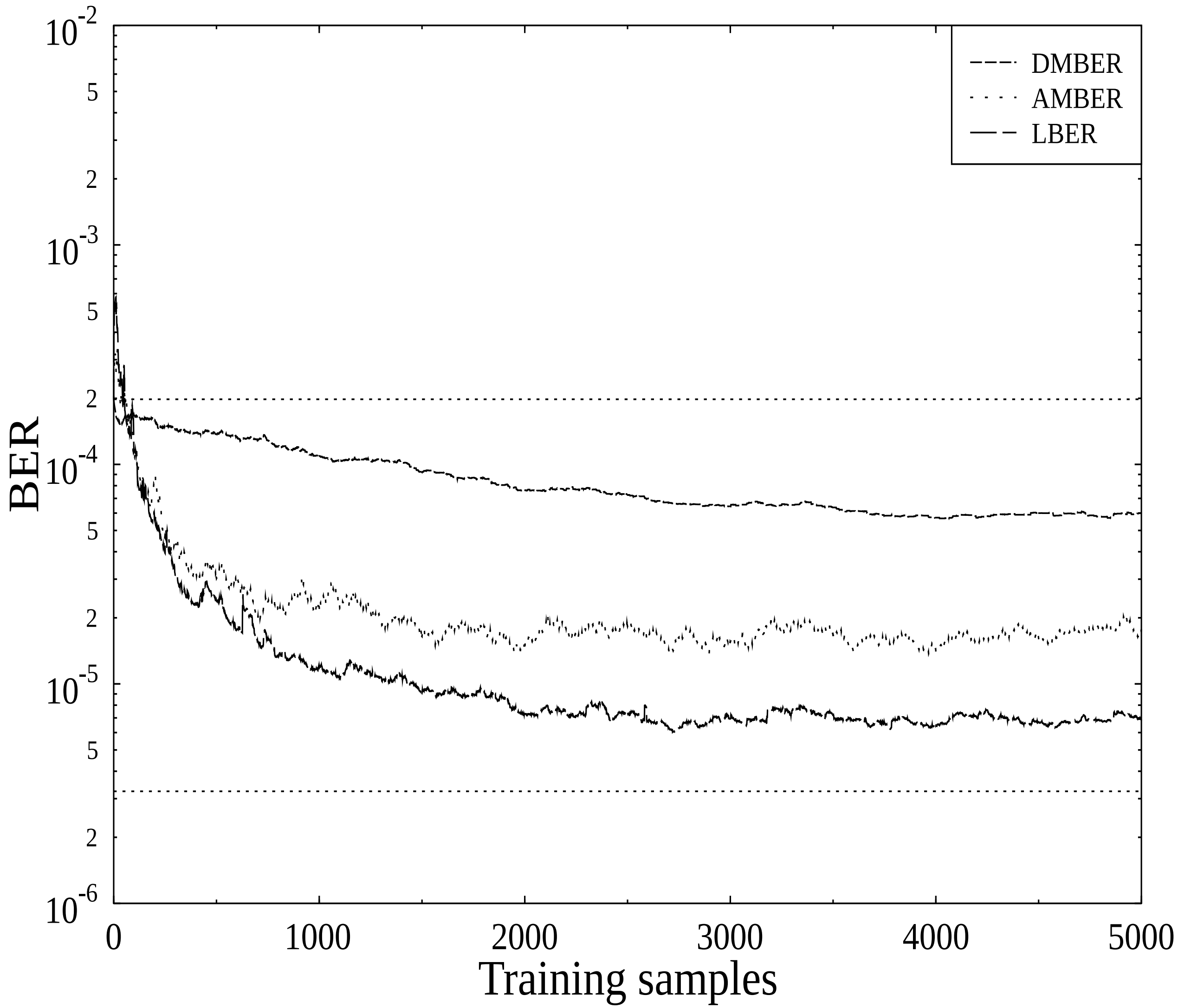




$\text{SNR}_i = 15$ dB for all i

Stochastic gradient adaptation for **user 1**:

Average over 50 runs



Conclusions

- MBER solution for linear multiuser detector can be superior over MMSE one
- LMS-style stochastic gradient adaptive MBER algorithms are available
- Our approach: Least Bit Error Rate, LBER
  - ⋆ Kernel density estimate for p.d.f. of detector decision variable is natural and generic[1]
  - ⋆ Complexity is linear with detector length
  - ⋆ Appear to have better performance in terms of convergence speed and steady-state BER misadjustment

[1]We have extended the LBER to training nonlinear neural network multiuser detectors