

MATH3084/MATH6162 Lecture plan 2022/23

1. **Complex variable theory** Overview of the course. Complex numbers **1-2.2**
2. Analytic functions, Cauchy-Riemann equations, Taylor series **2.3-2.4**
3. Examples of Taylor series, Laurent series, example **2.5**
4. Example: three different series expansions of $f(z) = 1/(2 - z^2)$ about $z = 1$
5. Classification of singularities, contour integration, integral of $(z - a)^n$ around a circle **2.6-2.7**
6. Cauchy theorem, proof on a rectangle, residue theorem and proof **2.8**
7. Cauchy integral formula, $N - P$ integral, winding number, argument principle **2.9**
8. Calculation of residues, examples **2.7**
9. **Contour integrals** Contour integrals to evaluate integrals over the real line **3.1**, ML estimates and examples **2.10**, example of semicircle contour
10. Multivalued functions **2.11**, examples
11. Integrals over the half-line: example of keyhole contour **3.5** (using $\ln z$), same example with pie-segment contour **3.6-**
12. Three more examples: box contour **3.3**, Fourier transform with semicircle contour **3.2**, trigonometric functions **3.8**
13. Revision of contour integration: semicircle contour, Fourier integrals, ML estimates, keyhole contour, pie segment contour, box contour, trigonometric functions **3.**
14. **Fourier transforms and δ -function** Derivation of Fourier transform (FT) from Fourier series, definition of FT and inverse FT, basic properties of FT, proof of convolution theorem **4.1-4.3**
15. Inverse Fourier transform of a Gaussian, integral of a Gaussian
16. The δ -function, relation between inverse FT and δ -function, relation between convolution and δ -function, Fourier transform pairs **4.4-4.6**
17. Application of FT to PDEs **4.7-4.8**, examples: wave equation in one space dimension, Laplace equation on the half-plane with Dirichlet BCs
18. Example: Poisson equation in three space dimensions (derivation of the Green's function)
19. Sine and cosine transforms (ST and CT) **4.9-4.10**
20. Application of ST and CT to PDEs **4.11**
21. **Laplace transforms** Laplace transform (LT): definitions and properties. Inverse LT via Bromwich contour, proof using inverse FT **5.1-5.3**
22. Evaluation of inverse LT using residue theorem, proof (with estimates), example $(2 - s)/(s^2 + 1)$ **5.4**
23. Two more examples: $1/(s \cosh as)$ and $1/\sqrt{s}$
24. Application of LT to PDEs. Two examples: transport equation on the line, wave equation on an interval. **5.5-5.6**
(Easter vacation)

25. Another example of LT applied to PDE: heat equation on an interval.
26. Hankel transform (HT) **6.1-6.3**
27. Relation between HT and FT **6.4**
28. Application of HT to PDEs **6.5**. Example: Laplace equation on a half space in cylindrical symmetry with mixed boundary conditions.
29. Nyquist theory **7**.
30. Example of Nyquist theory: $A(s) = (1 - s)/(1 + s)$, $B(s) = k/s$.