MATH3083/MATH6163

Advanced Partial Differential Equations, 2021/22

Module information

Content

If you have taken one of the prerequisite modules for this one (MATH2038, 2047, 2048 or 2015), you have seen a particular method for solving linear PDEs: separation of variables. Here we go further in three directions.

The material in Secs. 1-3 of the lecture notes (12 lectures) covers the crucial concept of well-posedness, and the classification of PDEs into types such as elliptic, parabolic and hyperbolic, based on their principal symbol.

Sec. 4 of the lecture notes (6 lectures) covers a type of hyperbolic PDE that is central in engineering, namely conservation laws. Here, the nonlinearity of the PDE takes centre stage, leading to discontinuous solutions. A key concept is the Riemann problem (piecewise constant initial data) and techniques for its solution.

Secs. 5-10 of the lecture notes (14 lectures) cover linear PDEs in typically three space dimensions, or three space dimensions and time. A key tool are Green's functions. If you think of the matrix equation $A\mathbf{y} = \mathbf{b}$, with solution $\mathbf{y} = A^{-1}\mathbf{b}$, as a toy model for the Poisson equation $\Delta\phi(\vec{x}) = f(\vec{x})$, then the Green's function is, in a certain sense Δ^{-1}. Before we introduce Green's functions for PDEs, we first introduce the δ-function and friends, and warm up with Green's functions for ODEs.

Lecturers

Professor Carsten Gundlach, c.j.gundlach@soton.ac.uk, Room 54/2017 (module lead, pre-recorded lectures), and Dr Michael Kenna-Allison, m.kenna-allison@soton.ac.uk, Room 54/5005 (in-person teaching).

Teaching materials

You will be given **full printed lecture notes** in the first tutorial session. The original print version will be on Blackboard, and also a version where we correct errors and make improvements as we go along (page numbers and equation numbers may change). There is a section of **exercises** at the end of each section of the notes, and numbered continuously across all sections. we will give you full solutions to these over the semester. There are **three revision exercises that you should attempt before the course starts**.

The module will be delivered through **30 pre-recorded lectures** (pen capture and voiceover) in the first 10 teaching weeks before Christmas. You will find those lectures on the "Pre-recorded lectures and recorded Sessions" tab on Blackboard. The **lecture plan** on Blackboard tells you which parts of the printed notes are covered in each lecture.

Some exercises in each section are additionally labelled as "**Homework** n". Each of these is designed to reenforce what you have just learned in Lecture n, and so you should attempt it right after that lecture. The homework problems will require both time and original thought, and are an integral part of the teaching.

Note that the MATH6163 Blackboard site will remain empty. All materials are on the MATH3083 site.

Getting help

Please ask your questions in the tutorials and problem classes! If there is something you do not understand, it is very likely you are not the only one, so it makes sense to ask in class. However, if you want to talk to one of us alone, either stay behind at the end of a live session, or email us to make an appointment.

Teaching methods

Our teaching method is sometimes called the "flipped classroom". You engage with the material on your own through the pre-recorded lectures, lecture notes, and homework problem. The interactive, in-person sessions are reserved for asking questions, while you work on your own. You get as much staff time as in the traditional three lectures plus one problem class model, but it is used differently.

To go with each lecture, there is an **online tutorial** each Monday, Tuesday and Wednesday, and there is **online problem class** each Friday. These should be shown on your timetable, and they will all be held in the "course room" on Blackboard Collaborate.

Please **watch each pre-recorded lecture before the corresponding tutorial.** A good plan, if you can fit it in, is to watch it on the same day, so it will be fresh in your mind for the tutorial. If taking notes in in-person lectures has helped you in the past, you should try doing this also in recorded lectures. You will probably need to stop or rewind the recording to take notes, think something through, or check a calculation.

The plan for the tutorials is that you **work on the homework problem** from the preceding lecture there and then, asking for help if and when you need. So T01 supports L01 and H01, and so on. We are hoping to create an experience similar to the First-Year Mathematics Workshop (MATH1046) or Engineering Mathematics Workshop (MATH1061). We will probably say a few words at the beginning of each tutorial, and then take any questions as they arise, talking to the whole class.

The tutorials will definitely be in person, but with a chance to participate online.

To encourage you to ask questions and talk to each other in the tutorials, and because there will be long silent periods, **we will not record the tutorials**. Hoever we will **save what we write on the electronic whiteboard** and put this up on the "Teaching materials" tab directly after each tutorial.

The plan for the problems that are not homework problems is that you use them for additional study or for revision. You can ask for help with any problem in any tutorial. We will give you some time to attempt them before releasing model answers.

Self-marking

To encourage you to engage with the course, and in particular with the homework problems, day by day, we will use self-marking. **The plan for the problem classes** is that we go over the three homework problems in that week, and that you mark your own work during the session. **At the time of writing we have not decided if the problem classes will be online or in person.** Either way you are still encouraged to ask questions! You should normally be able to self-mark your work during the problem class. If you had serious problems or errors, it may be more instructive to work that part of the problem again in your own time after the tutorial, and attach it to your original solution.

You will get 0, 1 or 2 marks per week. These are not for the correctness of your work, but for engaging with the material: **You will get the first mark for submitting a reasonable effort at a solution, and the second mark for accurate self-marking**. This means finding where you went wrong, and explaining to yourself what you should have done.

If you have not submitted something before the problem class, you can still get a single mark for submitting a correct complete solution by the self-marking deadline. The idea behind this is that you will learn something by attending the problem class and taking notes, although not as much as if you first try on your own. A typed model answer will then be released after the self-marking deadline.

Hence there are two deadlines for the first 10 teaching weeks before Christmas: **Friday ???? for your answers** to the three homework problems of that week (but you should try to submit on Thursday), and **Friday 23.59 for your self-marking** (but you should normally be able to submit right after the problem class).

Weekly workplan

In Week 0: Attempt the three revision problems, familiarise yourself with the structure of the course and the lecture notes, make contact with your group.

In Week 1:

Tuesday	Watch Lecture 1, read corresponding sections in the notes
	Attempt Homework 1, optionally attend Tutorial 1 **(????)**
Wednesday	Watch Lecture 2, read corresponding sections in the notes
	Attempt Homework 2, optionally attend Tutorial 2 **(????)**
Wednesday	Watch Lecture 3, read corresponding sections in the notes
	Attempt Homework 3, optionally attend Tutorial 3 **(????)**
Thursday	Consolidate and submit your homework answers
Friday	Attend Problem Class 1 **(???)**, self-mark your homework, submit again

Weeks 2-10 continue in the same way. **Weeks 11-12** are for revision: we will make a plan for that nearer the time.

Submission of weekly homework and your self-marking

In Week 1 at least, you will be asked to submit a scan of your handwritten solutions to H1-H3, mark them on paper using a red pen, and submit a scan of the marked homework. You will submit on Blackboard. Details later.

You must submit a scan of your work as a single PDF file, containing all pages the right way up and in the right order. On Blackboard there is a document written by James Vickers explaining how to do this on an Android phone. If you have a computer scanner or iphone, you probably know what to do.

Assessment

If you have already taken MATH3083, you cannot take MATH6163. Both modules have the same syllabus and homework problems, but the MATH6163 coursework and exam will have some harder questions.

20% of the final mark is from self-marking 30 homework questions, submitted and self-marked in 10 weekly chunks. These marks are for engaging with the material.

20% of the final mark is for two pieces of coursework. These will be designed to be the most challenging part of the assessment, to balance the easy 20% from the self-marking. Details later.

60% of the final mark is from the final exam. This will be online and open-book, and designed to be doable in two hours, although you will have an additional hour for scanning and uploading. Questions will be roughly similar to pre-covid years, but obviously any bookwork questions will be more subtle, and there will be more emphasis on accurate final results in calculations. All material in the printed notes is examinable except where announced. On Blackboard you will find all previous actual exams and two practice exams that from the the first year the course ran, all with model answers.

Books

PDEs is such a vast subject that there is no standard choice of material for a one-semester course. Therefore, there is no single book that covers the entire syllabus of this course, and that is why you have full lecture notes. On the other hand, there are lots of books on the topic, each with its own choice of material and emphasis. Have a browse in section QA374 of the library.

However, we would recommend the following two books for background reading. They have very different styles. Have a look at both in the library. Both go much wider and deeper than the lecture notes, of course.

M. Renardy and R. C. Rogers, *An Introduction to Partial Differential Equations*, Springer, 2nd edition 2004. (1st edition 1993 is also fine). QA374 REN.

J. Ockendon, S. Howison, A. Lacey and A. Movchan, *Applied Partial Differential Equations*, Oxford University Press, revised edition 2003. (1st edition 1999 is also fine.) QA377 OCK.

Chapter 1 of Renardy and Rogers is the best introduction to PDEs at the level of this course that I know. It starts from the solutions by separation of variables that you already know, and points out some important general ideas in these examples. I am trying to do the same in Sec. 3.1 of my lecture notes.

The rest of Sec. 3 of the lecture notes (classification and the symbol) loosely follows parts of Ch. 2 of Renardy and Rogers, also taking in ideas from Ch. 1-3 of Ockendon *et al.* The main difference is that for lack of time I am not introducing characteristics.

The best background reading for Sec. 4 of the lecture notes (conservation laws) would be R. J. Leveque, *Numerical Methods for Conservation Laws*, Birkhäuser 1992. QA377 LEV. Ch. 3 of Renardy and Rogers will also work.

Chs. 4, 5 and 6 of Ockendon *et al* can be used as background reading for Secs. 7-10 of the lecture notes (Green's function methods for linear elliptic, parabolic and hyperbolic equations).

Another two books are more elementary, and do not cover the whole syllabus, but are also useful.

Y. Pinchover and J. Rubinstein, *An Introduction to Partial Differential Equations*, Cambridge University Press 2005. QA374 PIN.

W. A. Strauss, *Partial Differential Equations, An Introduction*, John Wiley 1992. QA374 STR.