MATH3083/MATH6163

Advanced Partial Differential Equations, 2021/22

Lecture plan

Boldface and numbering denote sections in the printed notes. A * denotes material not or not completely covered in the printed notes. Sections shown in round brackets are for self-study, but are examinable unless also in *italics*.

1. **1 Overview**. 1.1.1 Partial derivative notation. (1.1.2 systems of PDEs), 1.1.3 PDE problems, 1.1.4 Linear PDEs, 1.2 Vector calculus, 1.3 Separation of variables), 1.4 Fourier series and Fourier transforms, 1.5.1 Laplace, Poisson and Helmholtz equations.
2. 1.5.2 Heat equation. 1.5.3 Wave equation.
3. 1.6 Cauchy data, Cauchy-Kowalewski. 1.7 Weak solutions of the 1D wave equation.
4. **2 Well-posedness**. 2.1 Definition of well-posedness. 2.2 Examples of ill-posed problems.
5. 2.3 Continuous dependence on the data.
6. . . . continued. 2.4.1 Energy estimate for the heat equation.
7. 2.4.2 Energy estimate for the wave equation. 2.4.3 Uniqueness of the boundary value problem for the Poisson equation. (*2.5 The importance of well-posedness.*)
8. **3 The symbol and classification** 3.1 Introduction. 3.2 The symbol.
9. 3.3 Ellipticity. 3.4 A model parabolic PDE.
10. 3.5 Hyperbolicity.
11. 3.6 Examples.
12. 3.7.1 First-order scalar PDEs. 3.7.2 Second order scalar PDEs. 3.8 Nonlinear PDEs.
13. **4 Conservation laws**. 4.1.1 1D conservation law: integral and differential form. 4.1.2 Higher space dimensions.
14. 4.2.1 The 1D advection equation. 4.2.2 Method of characteristics for 1D scalar conservation laws. 4.2.3 Propagation of small disturbances.
15. 4.3.1 Shock formation and Riemann problems.
16. 4.3.2 Propagating shock solutions.
17. 4.3.3 Rarefaction wave solution. 4.3.4 Lax condition. 4.3.5 Systems and higher dimensions.
18. 4.4 Traffic flow.
19. **5 Generalized functions**.
20. 5.5 Properties of the δ-function. **6 Green's functions for ODEs**. 6.1 First order.
21. 6.2 Harmonic oscillator. (6.3 General second order.) 6.4 Initial value problem.
22. **7 Green's functions for the Poisson and Helmholtz equations**. 7.1 Three dimensional δ-function. 7.2 Green's function for the Poisson equation on free space, derivation using the divergence theorem.
23. 7.3 Green's function for the Helmholtz equation. (*7.4 Derivation using differentiation under an integral.*) 7.5 Large distance and long wave-length approximation. (7.6 Uniqueness of the solution to the free-space problem.)

24. **8 Bounded regions**. (8.1 Green's theorem. 8.2 Reciprocal theorem.) 8.3 Kirchhoff-Helmholtz formula. 8.4 Problems on bounded regions.

25. 8.5 Method of images, two examples.

26. **9 The diffusion equation**. 9.1 Green's function for the 1D heat equation: dimensional analysis.

27. 9.2 1D heat equation: initial value problem, error function.

28. 9.3 Green's function for the 3D heat equation from 1D.

29. **10 The wave equation**. 10.1 One space dimension: 10.1.1 Green's function, 10.1.2 initial value problem.

30. 10.2 Three space dimensions: 10.2.1 Green's function, 10.2.2 retarded potentials. (*10.3 Method of descent, from 3D to 1D and 2D.*)