QUANTUM MECHANICS REVISION NOTES:

1 De Broglie Waves and Energies of Photons

De Broglie wavelength λ of a particle with momentum p

$$\lambda = \frac{h}{p}$$

or in terms of the wavenumber k

$$k \equiv \frac{2\pi}{\lambda} = \frac{p}{\hbar}.$$

Energy of a photon of frequency, ν (angular frequency ω),

$$E = h\nu = \hbar\omega$$

2 Interpretation of Wavefunction

The probability of finding a particle whose wavefunction of $\Psi(\mathbf{r})$, in a volume element d^3r at the position $\mathbf{r}$ is given by

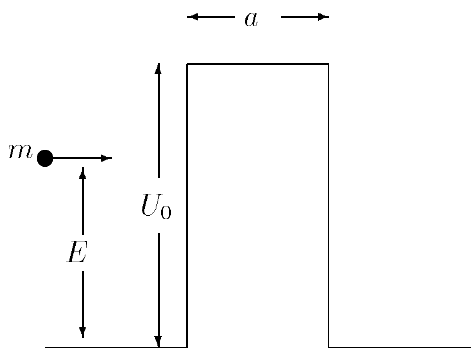
$$P(\mathbf{r})d^3r = |\Psi(\mathbf{r})|^2 d^3r$$

3 Free Particle Wavefunction

The wavefunction for a free particle moving in three dimensions, within a volume V with momentum $\mathbf{p}$ and energy E $(= p^2/(2m))$ is given by

$$\Psi(\mathbf{r}, t) = \frac{1}{\sqrt{V}} \exp\left\{i\left(\mathbf{p}\cdot\mathbf{r} - Et\right)/\hbar\right\}.$$

4 Quantum Tunnelling



Inside the barrier, where the potential energy is U_0, the kinetic energy is $E - U_0$ and so that the momentum, which is given by

$$p = \sqrt{2m(E - U_0)},$$

turns out to be imaginary ($= i\hbar\kappa$). Classically this does not make sense, but in quantum mechanics it means that the wavefunction is not an oscillatory function in this region, but an exponentially decaying function of position x. The transition amplitude the is ratio of value of the wavefunction on the right-hand edge of the barrier to the value of the wavefunction at the left-hand edge.

For a square potential of height U_0 and width a, the tunnelling amplitude for a particle with mass, m and energy E, is approximately given by

$$A = e^{-\kappa a},$$

where

$$\kappa = \sqrt{2m(U_0 - E)}\frac{a}{\hbar}.$$

The approximation is valid provided $(U_0 - E) \gg \hbar^2 m/a^2$.

The transition probability is $|A|^2$.

5 Harmonic Oscillator

The energy levels of a harmonic oscillator of angular frequency ω are

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega.$$

For a three dimensional harmonic oscillator they are given by

$$E_{n_1,n_2,n_3} \;=\; \left(n_1 + n_2 + n_3 + \frac{3}{2}\right)\hbar\omega.$$

6 Spherically Symmetric Potentials

The wavefunction of a particle moving in a spherically symmetric potential may be written (as a function of spherical polar coordinates)

$$\Psi_{n,l,m}(r,\theta,\phi) \;=\; R_{n,l}(r) Y_{L.m}(\theta,\phi)$$

where n is the principle quantum number
l is the angular momentum quantum number (i.e. $L^2 = l(l+1)\hbar^2$)
m is the magnetic quantum number (i.e. the z-component of angular momentum is $m\hbar$). The functions $Y_{l,m}(\theta,\phi)$ are called "spherical harmonics". They are functions of the angles only and depend on the quantum numbers l and m but *not* on the form of the spherical potential. The "radial function" $R_{n,l}(r)$ depends on the form of the potential.

The energy levels depend of n and l but not on m.
(For a Coulomb potential we have $n > l$, but this is *not* necessarily true for other potentials.)

7 Orbital Angular Momentum

The allowed eigenvalues of the operator L^2 are

$$l(l+1)\hbar^2$$

where l is a positive integer.

The eigenvalues of the z-component of angular momentum, L_z, are $m\hbar$, where m is an integer, which for a given l lies in the range

$$-l \;<\; m \;<\; l.$$

8 Expectation Value

The expetation value of some quantity, Q, for a system which is in a state i, is the average value of that quantity measured over a large number of identical systems each in the state, i.

If $\hat{Q}$ is the quantum-mechanical operator corresponding to the quantity, Q, then the expectation value is given by

$$\langle Q \rangle \;=\; \int \Psi_i^*(\mathbf{r}) \hat{Q} \Psi_i(\mathbf{r}) d^3\mathbf{r}.$$

9 Transition Amplitude

The amplitude A_{ji} for a transition to occur between a state with quantum numbers i and a state with quantum numbers j, due to a perturbing potential H' is given by

$$A_{ji} \;=\; \int d^3r \cdots \Psi_j^*(\mathbf{r},\cdots)H'\Psi_i(\mathbf{r},\cdots),$$

(where $\cdots$ allows for more than one particle). The transition rate for that transition is proportional to $|A_{ji}|^2$.