Advances in Homotopy Theory VII

Titles and Abstracts

Steven Amelotte

Title: Homotopy types of moment-angle complexes associated to almost linear resolutions

Abstract: Moment-angle complexes are central objects of study in toric topology which control the homotopy groups of all quasitoric manifolds. In this talk, we consider the problem of reading off the homotopy types of these spaces from homological properties of their associated Stanley-Reisner rings. We show that the Hurewicz image for any moment-angle complex contains the linear strand of its associated Stanley-Reisner ring. Combined with variants of a theorem of Eagon and Reiner well-known to commutative algebraists, we describe how this recovers results of various authors identifying the homotopy type of the moment-angle complex as a wedge of spheres when the Stanley-Reisner ring satisfies certain Cohen-Macaulay properties. Going further, we introduce a large class of Gorenstein simplicial complexes whose associated moment-angle manifolds have the rational homotopy type of connected sums of sphere products and the (integral) loop space homotopy type of products of loops on spheres.

This is joint work with Ben Briggs.

Aloke KR Ghosh

Title: $SU(2)$-bundles over highly connected 8-manifolds.

Abstract: In the case of simply connected 4-manifolds, the first kind of classification of circle bundles is the result of Giblin which states that the total space of a principal circle bundle over $M = S^2 \times S^2$, classified by a primitive class $\alpha \in H^2(M)$, is homeomorphic to $S^2 \times S^3$. We show an analogous result in the 8-dimensional case over $M = S^4 \times S^4$. We investigate from a homotopy theoretic point of view, which leads to study the possible homotopy type of the total space of a principal $SU(2)$-bundle over a 3-connected 8-dimensional Poincaré duality complex.

This is a joint work with Samik Basu and Subhankar Sau.

Jingyan Li

Title: On singular homology theories of digraphs and quivers

Abstract: Singular cubical homology theory can be constructed for different categories of digraphs and quivers based on two types of digraph cubes. Furthermore, there exist various categories of digraphs and quivers that share the same set of objects but have different sets of morphisms. Additionally, there are different definitions of homotopy, leading to the existence of several homotopy categories in graph theory. Similarly, singular simplicial homology theories can be defined on various categories (homotopy categories) of digraphs and quivers.

In this talk I will introduce the singular (h-singular) cubical homology group, the singular (h-singular) transitive cubical homology group, the singular (h-singular) simplicial homology group of quivers and digraphs and their functorial and homotopy properties.

Pengcheng Li

Title: Cohomotopy sets of manifolds of small dimension

Abstract: The k-th cohomotopy set of Given a based space X is the set of homotopy classes of based maps from X to the k-sphere. In this talk, we present some recent work on cohomotopy sets for manifolds of dimension ≤ 10. In case where the codimension arises, we impose additional condition that the manifolds are 2-torsion-free.

Toshiyuki Miyauchi

Title: Matrix Toda brackets in the EHP sequence

Abstract: H. Toda introduced the Toda bracket, a tool for computing the homotopy groups of spheres. In particular, the Toda bracket indexed by n is important in calculating the unstable homotopy groups of spheres using the EHP sequence. The matrix Toda bracket is a generalization of the Toda bracket introduced by M. G. Barratt. J. Yang, T. Miyauchi and J. Mukai (2024) defined the matrix Toda bracket indexed by n and obtained some relations between the matrix Toda bracket indexed by n and the EHP sequence. On the other hand, T. Miyauchi and J. Mukai (2017) introduced a definition different from the matrix Toda bracket indexed by n than that defined by Yang-Miyauchi-Mukai, and determined the 2-primary components of the 32-stem homotopy groups of spheres. The matrix Toda bracket indexed by n in these two definitions is different. In this talk, I will explain some properties of the matrix Toda bracket indexed by n defined by Miyauchi-Mukai and its relationship with the EHP sequence.

Aniceto Murillo

Title: Homotopy theory of complete Lie algebras

Abstract: We will present an overview of the recently developed homotopy theory of complete differential graded Lie algebras, extending the classical Quillen framework for rational homotopy theory. Particular focus will be given to key results that lie beyond the scope of the classical theory.

Victoria Oganisian

Title: Moment-angle manifolds corresponding to three-dimensional simplicial spheres and connected sums of products of spheres.

Abstract: The key question is to identify the class of simplicial spheres K for which the moment-angle manifold Z_K is homeomorphic to a connected sum of products of spheres. For two-dimensional spheres K, it is known that Z_K is diffeomorphic to a connected sum of products of spheres if and only if K is obtained from the 3-simplex by consecutively cutting off some l vertices. This characterisation can be extended by adding two more equivalent conditions, the chordality and the minimal non-Golodness.

For three-dimensional simplicial spheres K we prove the following: Z_K corresponding to a 3-dimensional simplicial sphere K has the cohomology ring isomorphic to the cohomology ring of a connected sum of products of spheres if and only if either:

(a) K is the boundary of a 4-dimensional cross-polytope, or

(b) the one-skeleton of K is a chordal graph, or

(c) there are only two missing edges in K and they form a chordless 4-cycle.

Moreover, if K is the nerve complex of four-dimensional simple polytope then under each of the conditions (b) and (c) above the moment-angle manifold Z_K is homeomorphic to a connected sum of products of spheres.

Guozhen Wang

Title: The last Kervaire invariant problem

Abstract: The Kervaire invariant is crucial for the classification of homotopy spheres. It is a long standing problem in which dimensions the Kervaire invariant is non-trivial. We will first review the history of the classification of homotopy spheres and the role of the Kervaire invariant. The we will give an acount on the solution of the last case of the Kervaire invariant problem in dimension 126.

Yunguang Yue

Title: Some applications of algebraic topology in distributed computing

Abstract: In this talk, I will give a brief introduction of some concepts on distributed computing and the framework of topology in distributed computing. Then I will introduce a kind of consensus tasks, that are loop tasks in low-dimensional situations and rendezvous tasks in high-dimensional situations. Lastly, I will show a classification of those special rendezvous tasks by algebraic topological method.