Analytical and numerical treatment of black holes in horizon-penetrating coordinates

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with

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Motivation

- Bridge gap between NR and perturbation theory $\rightarrow$ quantifying deviations important for waveform modeling
- QNM calculations are in Schwarzschild/Regge-Wheeler coordinates (Leaver, Andersson, Berti+) hence incompatible with horizon penetrating coordinates.
- Computation of the excitation amplitudes and tails from any initial configuration of the scalar field for all observers.
- Overtone excitation and detection for different types of initial data.
- Compare with numerical results using bamps (Brügmann, Hilditch+).

QNMs and the confluent Heun equation

- Solve $\Box\Phi = 0$ on a Schwarzschild background in Kerr-Schild
- $\Phi(t, r, \theta, \phi) = \sum_{l,m} K_{l,m}(t, r) Y_{l,m}(\theta, \phi)$.
- QNM boundary conditions ($\bar{\omega} = \omega M$):

$$K_{l,m} \sim \frac{1}{r} e^{-i\omega(t+r)}, r \to 2M, \quad K_{l,m} \sim \frac{1}{r} e^{-i\omega(t-r)} \left(\frac{r}{2M}\right)^{4i\bar{\omega}}, r \to \infty.$$

- Frequency domain Green's function (for single l):

$$G_{l,m}(\omega, r, r') = \frac{p(\omega, r')}{A(\omega)} \begin{cases} f_-(\omega, r) f_+(\omega, r'), & r \leq r', \\ f_-(\omega, r') f_+(\omega, r), & r' \leq r, \end{cases}$$

  where $p = r^2 (r - 2M)^{-4i\omega M}$ and $A = w\left(f_- f_+' - f_-' f_+\right)$.
- $f_\pm = e^{-i\omega r} H(r/2M)$ with H being two linearly independent solutions of the confluent Heun equation (Heun, Ronveaux, Fiziev), $r = 2Mx$:

$$\frac{d^2 H(x)}{dx^2} + \left(\alpha + \frac{\beta + 1}{x} + \frac{\gamma + 1}{x - 1}\right) \frac{dH(x)}{dx} + \left(\frac{\mu}{x} + \frac{\nu}{x - 1}\right) H(x) = 0.$$

- Using analytic continuation (Slavyanov+, Philipp+) and U-series solutions (Leaver):

$$f_- = \frac{1}{2Me^{4i\bar{\omega}}} e^{i\omega r} \left(\frac{r}{2M}\right)^{-1+4i\bar{\omega}} \sum_{n=0}^{\infty} a_n \left(\frac{r-2M}{r}\right)^n ,$$

$$f_+ = \frac{1}{2M(-4i\bar{\omega})^{-1+4i\bar{\omega}}\Gamma(1-4i\bar{\omega})} e^{i\omega r} \sum_{n=0}^{\infty} a_n \Gamma(1+n-4i\bar{\omega}) U(1+n-4i\bar{\omega}, 1, -2i\omega r).$$

- Both $\{a_n\}$ $\rightarrow$ same three term recurrence $\rightarrow$ QNMs continuous fraction equation $\rightarrow$ same result as Leaver!
- Initial data evolves according to:

$$K_{l,m}(t,r) = \int G_{l,m}(t,r,r')\partial_t K_{l,m}(0,r')dr' + \int \partial_t G_{l,m}(t,r,r')K_{l,m}(0,r')dr'.$$

- Calculate separately individual contribution of poles and branch cut:

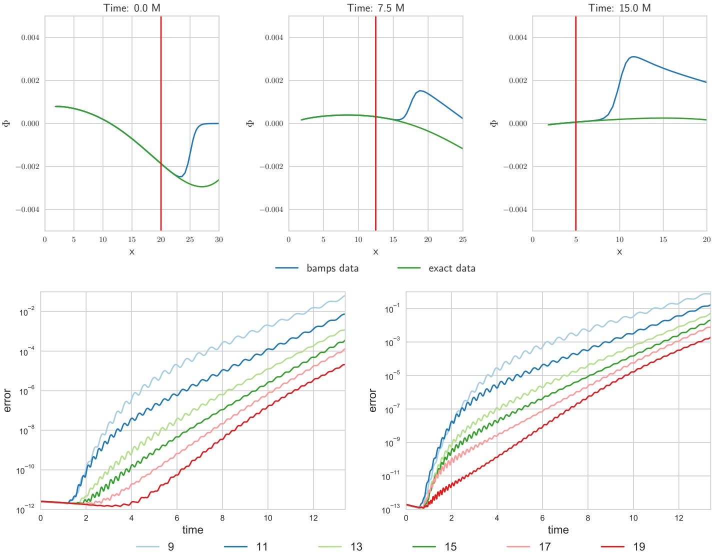
$$G_{l,m} = G^Q_{l,m} + G^B_{l,m} + \dots$$

- Respect causality! Integration limits by solving:

$$r' + 4M\log(r'-2M) = r + 4M\log(r-2M) - t,$$

and $r' = r + t$ for the upper limit.

Exact solution tests



- Very accurate QNM extraction: $0.11043074 - 0.10485913i$ ($< 0.01\%$ error) for $n = 0$ and $0.0857 - 0.3472i$ ($< 0.1\%$ error) for $n = 1$.

Branch cut contribution

- Branch cut contribution:

$$G^B(t,r,r') = \frac{1}{2\pi}\int_0^{-i\infty} p(\omega,r')f_-(\omega,r')\left[\frac{f_+(\omega e^{2\pi i},r)}{A(\omega e^{2\pi i})} - \frac{f_+(\omega,r)}{A(\omega)}\right] e^{-i\omega t} d\omega.$$

- Simplify expression for asymptotic observers $\rightarrow$ use Whittaker solutions.
- Late times $\rightarrow$ low frequency expansion of Whittaker functions $\rightarrow$ BesselJ!
- Approximate Green's function for asymptotic observers:

$$G^B(t,r,r') \approx C_1 \frac{r^l r'^{l+2}}{\eta(t,r,r')^{2l+3}} F_\Lambda\left(\{l\}; -\frac{2r}{\eta(t,r,r')}, -\frac{2r'}{\eta(t,r,r')}\right), \eta = t - f_1(r,r').$$

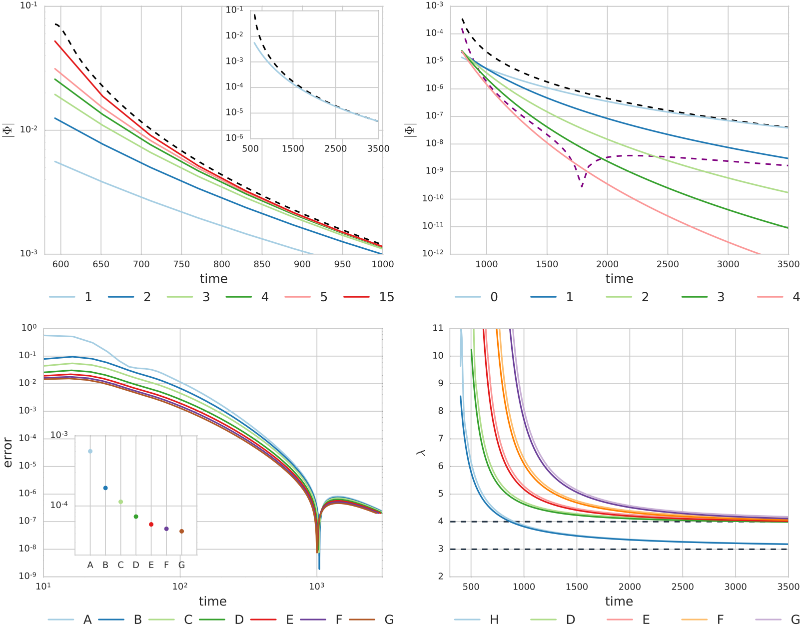
- Alternative expression:

$$G^B(t,r,r') \approx \sum_{m=0}^{\infty} \frac{C_2 r'^{l+2} r^{l+2m}\Gamma(2l+2m+3)\,{}_2F_1\left(\{l,m\}; \frac{r'^2}{r^2}\right)}{m!\Gamma\left(l+m+\frac{3}{2}\right)\left(t - f_2(r,r')\right)^{2l+2m+3}}$$

- Obtain familiar power law at late times:

$$G^B \sim \frac{1}{t^{2l+3}}.$$

Tail tests



QNM excitation factors

- QNM contribution to Green's function:

$$G^Q(t,r,r') = \sum_{l=0}^{\infty}\sum_{n=0}^{\infty} B_{l,n} e^{-i\omega_{l,n} t} \begin{cases} p(\omega,r') f_-(\omega_{l,n},r) f_+(\omega_{l,n},r'), & r \le r', \\ p(\omega,r') f_-(\omega_{l,n},r') f_+(\omega_{l,n},r), & r' \le r. \end{cases}$$

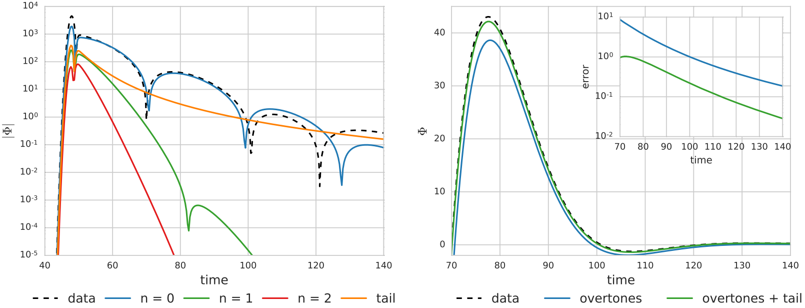
- Can now calculate excitation amplitudes for all observers.
- Assume near the poles:

$$A(\omega_{l,n}) \approx (\omega - \omega_{l,n}) A'(\omega_{l,n}).$$

- Some QNEfs:

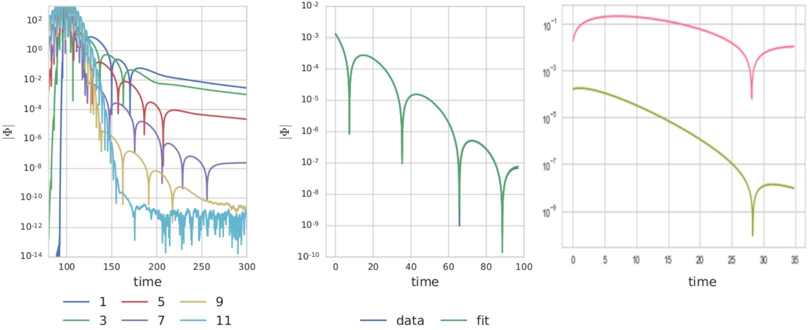
l	n	$A'(\omega_{l,n})$	$B_{l,n}$
0	0	$1.32962 + 3.01240i$	$0.55566 + 0.24526i$
	1	$4.37158 + 0.92283i$	$0.09244 + 0.43798i$

Results



- Overtones become more important at earlier times $\rightarrow$ $n = 1$ just an order of magnitude less than $n = 0$ near start.
- Summing overtones not sufficiently good at intermediate and late ringing.
- Late time tail result works remarkably well for earlier times $\rightarrow$ error becomes one order less at intermediate and late time ringing.
- Need contribution from high frequency arc to explain the beginning $\rightarrow$ unfortunately not so easy!

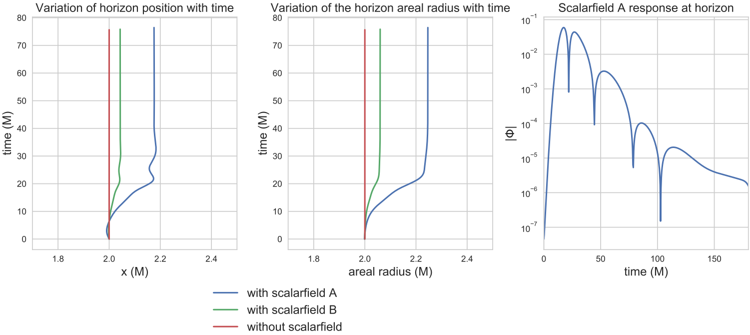
Results



- $l = 0$ ringdown short in general $\rightarrow$ branch cut contribution dominates after a few cycles
- $n = 0$ dominates for all ID $\rightarrow$ can recover ω_{QNM} within $\sim 1\%$ error.
- Longer ringdown with sine-Gaussian ID $\rightarrow$ more reliable ω_{QNM} extracted ($\sim 0.03\%$)
- $n = 1$ for sine-Gaussian ID $\rightarrow$ larger errors ($\sim 10\%$).

Ongoing work

- Solve the conformal transverse traceless form of constraints for Ψ and $\bar{V}^i$.
- Hyperbolic relaxation (Rüter+): $\partial_t^2\psi + \partial_t\psi = \Delta\psi$
- Conformal quantities $\rightarrow$ Schwarzschild quantities in Kerr-Schild
- Evolve using generalized harmonic coordinates (Lindblom+)
- Event horizon locator: Integrate 'outgoing' null geodesic backwards (Bohn+)



- Excision fails for 'large' perturbations $\rightarrow$ fix!

Thanks!