Deadline and welfare effects of scheduling information releases

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Abstract

How should institutions convey relevant information to the public? Should they schedule their communications or release information as it becomes available? What are the welfare effects of an unanticipated information release? We model a decentralized economy and show that a credible schedule delays trade towards the information release date and unanticipated information arrivals entail a loss of insurance opportunities. We apply these findings to the scheduling of monetary policy decisions following the Federal Open Market Committee meetings from 1995 till 2010 and its effects on the dynamics of trade on the Federal Funds Futures market. We use the model to empirically identify periods of credible (prior to 2001) and non-credible scheduling (after 2001). Finally we measure the loss in risk-trading activity due to off schedule announcements.

Keywords: Deadline effect, Hirshleifer effect, search in financial markets, monetary announcements, interest rate futures.
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1. Introduction

The value of public information is among the fundamental questions in economics and finance. A strictly related issue is how public and private agencies should convey information

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to the public. Should these agencies schedule their communications or release the information as it becomes available? Once a schedule is in place, how important is the credibility of the procedure? Today, many public institutions release information according to a schedule of announcements: this is the case of central banks announcing target interest rates at the end of the respective monetary policy committee meetings and of governmental agencies, like the Bureau of Labor Statistics, responsible for the official release of economic statistics on unemployment and inflation.

Little consideration has been given to the effects of such scheduling on the trade dynamics and on welfare. The theoretical contribution of this paper is to show that observables like trade volume dynamics can identify welfare effects of credible scheduling. In particular we will address the following questions: 1) Does scheduling communications change the dynamics of trade and if so how? 2) What are the welfare costs of "spontaneous" deviations from the schedule *i.e.*, off schedule announcements? 3) Can the reliability of the schedule be inferred from the dynamics of trade?

These issues are first analyzed in a theoretical model. We then apply the model to identify the effects of the Federal Open Market Committee (FOMC) scheduled and off schedule monetary policy announcements on the trading volume of Chicago Board of Trade (CBOT) 30-Day Federal Funds Futures.

The theoretical model studies financial markets under uncertainty where traders have access to two financial instruments: a risk-free bond and a risky asset. The risk-free bond is exchanged in a centralized market. The risky asset is exchanged over-the-counter according to a dynamic matching procedure where at each point in time buyers and sellers meet and declare their reservation price: if an agreement is reached they split the gains from trade equally and leave the market; otherwise they continue their search for a counterparty. Uncertainty is resolved by a public announcement reaching the market at some future date that might or might not be known to the traders. The announcement reveals the state of the world determining the risky asset return and the agents' endowment at the end of the economy. Agents are symmetrically informed about both the realization and the timing of

the announcement as in the case of many public announcements like interest rates decisions. In the first instance, the model analyzes the dynamics of trade when agents know the *exact date* the uncertainty will resolve (credible schedule); subsequently the model studies the case when the exact date is unknown to the agents (stochastic announcements).

We can summarize the results of the theoretical model as follows: 1) if (and only if) agents are risk averse, scheduling the communications changes trade dynamics by delaying a large volume of transactions towards the announcement date. This is the deadline effect of credible scheduling; 2) when risk averse agents exchange the asset in order to hedge uncertainty, spontaneous deviations from the schedule might be welfare impairing as once uncertainty is resolved risk sharing opportunities are lost. This is as in Hirshleifer (1971)[1]. 3) with stochastic announcements (it suffices a small but positive probability of an off schedule intervention), both deadline and welfare effects vanish.

The crucial insight of the analysis is that perfectly anticipated future arrivals of payoff-relevant public information act as a trading deadline for risk averse investors: when the schedule is reliable, traders act *as if* they had a limited time to exchange the risky asset as, once uncertainty is resolved, this becomes redundant and ceases to be a hedging instrument. We apply these theoretical findings to the FOMC monetary policy scheduling from January 1995 to July 2010 and look at the impact on the dynamics of trade of the CBOT 30-Day Federal Funds Futures market. We first study the deadline effect by checking if the volume of transactions is higher the days before a scheduled meeting. Our empirical analysis shows a statistically significant deadline effect for meetings prior to September 2001. We identify this split in the data set by employing rolling windows of 400 trading days with 60 days of overlapping gap. The two periods have a different monetary policy scheduling credibility: high till 2001 and less so afterwards. Unfortunately our model does not shed light on the reasons causing this shift in credibility.

We then turn to the evidence on the welfare effect. Interest rate futures are traded over-the-

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[1]Notice that our model, for tractability reasons, will focus on the Hirshleifer effect only and will not account for the other benefits of receiving the information earlier rather than later (the "Blackwell effect" after Blackwell (1951)). On the comparison of the two effects see Gottardi and Rahi (2008).

counter in order to hedge against changes in the target rate. If monetary policy scheduling is credible, an unanticipated resolution of uncertainty, due to an off schedule announcement, could be welfare impairing in the sense pointed out by Hirshleifer (1971). Although a substantial theoretical literature has extended and qualified Hirshleifer's result and identified sufficiently general conditions for the argument to hold[2], the empirical evidence is at best sparse[3], probably due to the difficulty in identifying instances where risk averse traders are "taken by surprise" by the earlier resolution of uncertainty. It seems natural to ask whether the welfare effect pointed out by Hirshleifer is a purely theoretical conjecture with little empirical relevance or, to the contrary, there are important instances where we observe its occurrence. If so what is the magnitude of the loss, either in terms of welfare or of trading volumes?

The off schedule FOMC meetings, all occurring in the period identified as of credible scheduling, provide the opportunity for running a natural experiment on the effects of monetary policy surprises. We first argue that these events were indeed unanticipated by showing that there was no significant excess trading activity before their occurrence. We then quantify the missed expected volume of trade following these surprises.

Modeling financial markets with decentralized mechanisms is not new and these are empirically relevant in the interest rate futures market. The search theoretic literature on financial markets includes Duffie, Gârleanu and Pedersen (2005) and (2007), Miao (2006) and Rust and Hall (2003), Lagos and Rocheteau (2007) and (2009). A distinctive feature of our case is that trade occurs under deadlines and hence we cannot analyze steady states.

Section 2 presents the theoretical model and the results. Section 3 presents the empirical model and the analysis. Section 4 concludes. With the exception of Theorem 1, all proofs

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[2]With different degrees of generality, Marshall (1974), Green (1981) and Hakansson *et al.* (1982) identify cases where a partial increase of information cannot be Pareto improving. Wilson (1975) shows that better information is Pareto impairing when agents have preferences represented by a log utility function. More recently, Schlee (2001) has given general conditions guaranteeing that public information is Pareto impairing and Eckwert and Zilcha (2003) have showed that information referring to tradable assets might be undesirable if agents are enough risk averse. Finally Gottardi and Rahi (2008) have provided sufficient conditions on the degree of market incompleteness for the information to have social value. For further discussion of the literature see Schlee (2001) and Gottardi and Rahi (2008).

[3]The only pieces of evidence we are aware of come from medical studies, in particular Lerman *et al.* (1996) and Quaid and Morris (1993).

can be found in the appendix.

2. Description of the economy

We consider an infinite horizon, one-good economy under uncertainty, extending over time $t \in [0, 1]$.

The economy is populated by the traders and the information provider. Traders have access to two financial instruments: a risk-free bond and a risky asset. The bond is traded in competitive markets, is perfectly divisible, its net supply is zero and pays one unit of the good after the state has realized. The asset is traded over-the-counter (OTC), is indivisible and offers a stochastic return $\rho(\cdot) : \Sigma \to \mathbb{R}_+$ payable to the asset holder at $t = 1$. Traders are partitioned into two types, denoted by $a = b, s$: the buyers, $a = b$, holding no assets; the sellers, $a = s$, holding one unit of the asset. There is a continuum of agents on each side of the market. Buyers and sellers meet according to the mechanism we will describe later and, for simplicity, they exchange one, indivisible unit of the asset: this is for tractability but not unrealistic in the case of trade of contracts of large size as interest rates futures.

At $t = 0, 1$ each agent receives a stochastic and agent-specific endowment $\omega_t(\cdot) : \Sigma \to \mathbb{R}_+$ with $\mathbb{R}_+$ the support of the distribution.

The only role of the information provider is to announce the state of the world. We will start by analyzing the game when the announcement date is common knowledge among the players, fixed for convenience at $t = 1$, and fully credible. In section 2.7 we will analyze the case where the announcement is stochastic, *i.e.,* where the information provider releases of information at any other $t \in (0, 1)$ with positive probability.

The good is non-storable. Consumption takes place right after trading, denoted by x_0 and after the state has realized, denoted by x_1. Agents' preferences are time and state separable, and represented by the same utility function:

$$u(x_0) + Eu(x_1),$$

Notice that there are no transaction costs nor costs of holding the asset between the date

the trade occurs and the realization of the state.

Assumption 1. *The utility function $u(\cdot)$ is strictly increasing and concave.*

For tractability we assume that the bond market opens at $t = 1$, once the trade in the OTC market has been exhausted and before uncertainty is realized. Agents decide how much to buy/sell of the asset and then how many units of the bond to hold. Since the structure of the economy is common knowledge, agents can compute the price the bond will trade at and will take this into account when deciding the asking and bidding price of the asset.

2.1 The OTC matching process

We discretize the time interval $[0,1]$ by partitioning it into $L+1$ subperiods, $l = 1, ..., L$, each of length $\epsilon = (L+1)^{-1}$. Each $t = l\epsilon$ denotes a trading session in the OTC market. Trade does not take place at $t = 1$ and between trading sessions. Let $\mathcal{T}_\epsilon = \{\epsilon, .., l\epsilon, ..., L\epsilon\}$ denote the set of trading sessions.

At session $t \in \mathcal{T}_\epsilon$ each agent is characterized by his type, the endowment ω_0 and the history $h_t \in H_t = \{in, out\}$. The history parameter h_t takes value *in* when a trader is active in the market. Each active buyer is matched randomly to one active seller. Once the match occurs they contemporarily declare their reservation price. If the bid is greater or equal to the ask price, exchange takes place, agents consume and their history takes value *out*. Buyers and sellers that fail to exchange proceed to the next trading session where (almost surely) they will meet a different counterparty. At each trading session the agents also decide how much of the bond to purchase on the competitive market. The agents not able to trade by the last session $t = \epsilon L$ trade the bond and consume before the state of the world is revealed.

Consider now any pair of traders and let b and s represent bid and ask prices for the buyer and seller, respectively. The price at which each pair exchanges is determined by a pricing rule $M(b, s)$ with the following properties:
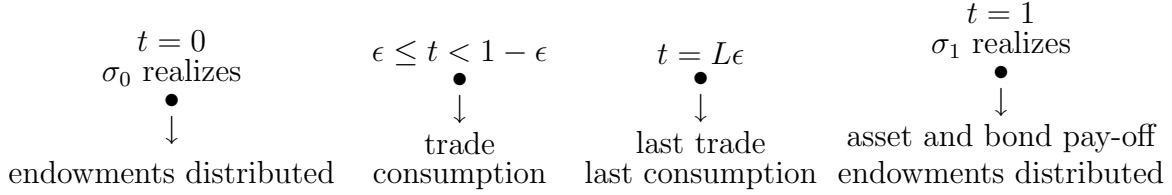
Assumption 2. *For each trading session:*
a) $M(b, s)$ is continuous in b and s;

b) if $b = s$ then $M(b, s) = b = s$.

Without loss of generality to our subsequent analysis we shall consider $M(b, s) = \frac{1}{2}(b + s)$[4].

For a given $\epsilon > 0$, the timeline can be described as follows:

$t = 0$ σ_0 realizes	$\epsilon \le t < 1 - \epsilon$	$t = L\epsilon$	$t = 1$ σ_1 realizes
•	•	•	•
↓	↓	↓	↓
endowments distributed	trade consumption	last trade last consumption	asset and bond pay-off endowments distributed

2.2 The $\mathcal{G}_\epsilon$ game

Fix an $\epsilon > 0$. At any trading session $t \in \mathcal{T}_\epsilon$ a buyer with endowment ω_0 and bid b_t meeting a seller with ask price $x \in (0, b_t]$ and holding portfolio y_t obtains utility $u(\omega_0 - \frac{b_t + x}{2} - q_\epsilon y_t) + Eu(\omega_1 + \rho + y_t)$, where q_ϵ denotes the price of the bond when the frequency of trade in the OTC market is ϵ and y_t the agent's bond's holding at session t. Similarly, a seller with total wealth ω_0 and ask price s_t meeting a buyer with bid $x \in [s_t, \infty)$ and holding portfolio y_t obtains utility $u(\omega_0 + \frac{s_t + x}{2} - q_\epsilon y_t) + Eu(\omega_1 + y_t)$. We assume preferences and endowments' distribution to be common knowledge so that agents can compute the distribution of bid and ask prices in order to maximize their expected utility.

Therefore, the ϵ-step optimization problem at $t \in \mathcal{T}_\epsilon$ for agent of type $a = b, s$ active in the market is given by:

$$V_\epsilon^b(t, in; \omega_0) = \max_{b_t} \int_0^{b_t} [u(\omega_0 - \frac{b_t + x}{2} - q_\epsilon Y_{\epsilon,t}) + Eu(\omega_1 + \rho + Y_{\epsilon,t})] dF_{\epsilon,t}^s(x)$$

$$+ (1 - F_{\epsilon,t}^s(b_t)) V_\epsilon^b(t + \epsilon, in; \omega_0), \tag{1}$$

$$Y_{\epsilon,t} \in \arg\max_{y_t} u(\omega_0 - \frac{b_t + x}{2} - q_\epsilon y_t) + Eu(\omega_1 + \rho + y_t) \text{ for all } x < b_t;$$

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[4]Different trading rules may satisfy Assumption 2. For example if the buyer (seller) makes a take-it-or-leave-it offer, then $M(b, s) = b$ ($M(b, s) = s$). If the proposer (buyer or seller) is chosen randomly with equal probability then one obtains $M(b, s) = \frac{1}{2}(b + s)$ as in Gale (1986). The bargaining can have a Nash solution at each t such that $M(b, s) = zb_t + (1 - z)s$ where the weight z is exogenously given as in Duffie, Gârleanu and Pedersen (2005) and (2007).

$$V_\epsilon^s(t, in; \omega_0) = \max_{s_t} \int_{s_t}^\infty [u(\omega_0 + \frac{s_t + x}{2} - q_\epsilon Y_{\epsilon,t}) + Eu(\omega_1 + Y_{\epsilon,t})]dF_{\epsilon,t}^b(x)$$

$$+ F_{\epsilon,t}^b(s_t-)V_\epsilon^s(t + \epsilon, in; \omega_0), \tag{2}$$

$$Y_{\epsilon,t} \in \arg\max_{y_t} u(\omega_0 + \frac{s_t + x}{2} - q_\epsilon y_t) + Eu(\omega_1 + y_t) \text{ for all } x > s_t,$$

where $V_\epsilon^a(t, in; \omega_0)$ is the value function at t for the agent with endowment ω_0, $F_{\epsilon,t}^s(b_t)$ is the proportion at t of sellers willing to sell for a price less than b_t, $F_{\epsilon,t}^b(s_{\epsilon,t}-)$ is the proportion of buyers at t willing to buy for a price strictly less than $s_{\epsilon,t}$ and finally q_ϵ is the price of the asset when the trading frequency in the OTC market is ϵ. Between any two consecutive trading sessions t and $t + \epsilon \in \mathcal{T}_\epsilon$ (i.e., the interval $(t, t + \epsilon)$) the agents' value functions are fixed at $V_\epsilon^a(t + \epsilon, in; \omega_0)$. The same holds for agents' history, bid and ask prices and their distribution, all being held constant in the interval $(t, t + \epsilon)$ at their $t + \epsilon$ value.

Once the last trading opportunity has elapsed and before the returns are distributed the value function for an agent that could not find a match is given by:

$$V_\epsilon^b(1, in; \omega_0) = max_{y_1}(\omega_0 - q_\epsilon y_1) + Eu(\omega_1 + y_1), \tag{3}$$

$$V_\epsilon^s(1, in; \omega_0) = max_{y_1} u(\omega_0 - q_\epsilon y_1) + Eu(\omega_1 + \rho + y_1). \tag{4}$$

Notice that the problems defined in equations (1) and (2) are non-stationary. Equations (3) and (4) give the terminal values which prevents the value function to be unbounded.

The optimal trading strategies for the buyer and seller with endowment ω_0 are represented by the vectors $(Y_{\epsilon,t}^b(\omega_0), B_{\epsilon,t}(\omega_0)) : H_t \rightarrow \Re_+^2 \cup \emptyset$, and $(Y_{\epsilon,t}^s(\omega_0), S_{\epsilon,t}(\omega_0)) : H_t \rightarrow \Re_+^2 \cup \emptyset$, respectively. History taking value *out* is mapped into the empty set. For notational convenience we will avoid the reference to the endowment when writing bond holdings and trading strategies, and will denote them as $Y_{\epsilon,t}, B_{\epsilon,t}$ and $S_{\epsilon,t}$, respectively. Given a set of trading sessions $\mathcal{T}_\epsilon$, the $\mathcal{G}_\epsilon$ game is specified by the array:

$$\mathcal{G}_\epsilon = \langle \Omega, \ V_\epsilon^a(t, h_t; \omega_0^a), a_t, \mathcal{H}_{\epsilon,t}, F_{\epsilon,t}^a, Y_{\epsilon,t}^a \ a = b, s, t \in \mathcal{T}_\epsilon \rangle.$$

where $\Omega_0 = \{\omega_0\}$ is the set of all agents and $\mathcal{H}_t : \Omega_0 \rightarrow \prod_t H_t$ is the set of all feasible

histories for all agents in $\mathcal{G}_\epsilon$ at time t.

In the appendix we provide a definition of the (subgame perfect) equilibrium for $\mathcal{G}_\epsilon$ and sketch a proof of the existence. However, our results on the volume of trade will follow from the characterization of individuals' trading strategies and not from the characterization of the equilibrium behavior. Unless necessary for clarity of exposition and in order to simplify notation we will drop the reference to the history in the value function.

The following lemma shows that the optimal bond holding decision does not depend on the time at which trade in the asset occurs and on the frequency of the sessions. This property substantially simplifies the analysis of the dynamics of portfolio holding.

Lemma 1. *The optimal bond holding is t and ϵ-independent, i.e., $Y_{\epsilon,t} = Y$ for all $t \in \mathcal{T}_\epsilon$. The bond's price and the last trading session value function are ϵ-independent i.e., $q_\epsilon = q$ and $V_\epsilon^b(1, in; \omega_0) \equiv \overline{V}^a(\omega_0)$, $a = s, b$.*

Proof of Lemma 1: See Appendix.

We shall show the dynamics of both bids and ask prices is monotonic. Substituting the solution $S_{\epsilon,t}$ in the seller's problem (2) obtain:

$$V_\epsilon^s(t; \omega_0) = \int_{S_{\epsilon,t}}^\infty [u(\omega_0 + \frac{S_{\epsilon,t} + x}{2} - qY) + Eu(\omega_1 + Y)]dF_{\epsilon,t}^b(x) \tag{5}$$
$$+ F_{\epsilon,t}^b(S_{\epsilon,t}-)V_\epsilon^s(t + \epsilon; \omega_0).$$

This can be written as:

$$V_\epsilon^s(t; \omega_0) - V_\epsilon^s(t + \epsilon; \omega_0) \tag{6}$$
$$= \int_{S_{\epsilon,t}}^\infty [u(\omega_0 + \frac{S_{\epsilon,t} + x}{2} - qY) + Eu(\omega_1 + Y) - V_\epsilon^s(t + \epsilon; \omega_0)]dF_{\epsilon,t}^b(x).$$

Since the individuals are willing to trade then:

$$u(\omega_0 + \frac{S_{\epsilon,t} + x}{2} - qY) + Eu(\omega_1 + Y) \geq V_\epsilon^s(t + \epsilon; \omega_0), \tag{7}$$

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for all $x \in [S_{\epsilon,t}, \infty)$ and $\epsilon > 0$.

This implies that $V_\epsilon^s(t; \omega_0)$ is a monotone decreasing function in t and therefore continuous except for countable many points. A similar argument holds for the buyer.

Letting $\lim_{\epsilon \to 0} V_\epsilon^a(t + \epsilon; \omega_0) \equiv V^a(t; \omega_0)$, for $a = b, s$ it follows that the function $V^a(t; \omega_0)$ for $a = b, s$ is L^p integrable with respect to t and is finite[5].

2.3 The deadline effect

We are now in the position to prove the main result on the dynamics of trade in the presence of a known deadline at $t = 1$. We shall analyze the game for frequent enough trading sessions, i.e., $\epsilon \to 0$ (or, equivalently, $L \to \infty$). From (6) it follows that:

$$
\begin{aligned}
& V_\epsilon^s(t; \omega_0) - V_\epsilon^s(t + \epsilon; \omega_0) \\
& = \int_{S_{\epsilon,t}}^\infty [u(\omega_0 + \frac{S_{\epsilon,t} + x - qY}{2}) + Eu(\omega_1 + Y) - V_\epsilon^s(t + \epsilon; \omega_0)] dF_{\epsilon,t}^b(x) \\
& \geq \int_{S_{\epsilon,t}}^\infty [u(\omega_0 + S_{\epsilon,t} - qY) + Eu(\omega_1 + Y) - V_\epsilon^s(t + \epsilon; \omega_0)] dF_{\epsilon,t}^b(x) \qquad (8) \\
& = [u(\omega_0 + S_{\epsilon,t} - qY) + Eu(\omega_1 + Y) - V_\epsilon^s(t + \epsilon; \omega_0)] \left(1 - F_{\epsilon,t}^b(S_{\epsilon,t})\right).
\end{aligned}
$$

From (8) it follows that for a given trading session $t \in \mathcal{T}_{i,\epsilon}$:

$$
\begin{aligned}
& \lim_{\epsilon \to 0}[V_\epsilon^s(t; \omega_0) - V_\epsilon^s(t + \epsilon; \omega_0)] \\
& \geq \lim_{\epsilon \to 0}[u(\omega_0 + S_{\epsilon,t} - qY) + Eu(\omega_1 + Y) - V_\epsilon^s(t + \epsilon; \omega_0)] \left(1 - F_{\epsilon,t}^b(S_{\epsilon,t})\right).
\end{aligned}
$$

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[5]REMARK: By Lusin's Theorem (p. 230, Billingsley (1986)), $V^s(t; \omega_0)$, can be approximated arbitrarily close by a continuous function. This implies that the set of ϵ-step sellers' problems defined by (2) for which the value functions $V^s(t; \omega_i)$ are continuous in t are dense in the set of all ϵ-step sellers' problems. Hence the set of problems for which the continuation values are discontinuous is negligible. Similarly, the set of ϵ-step buyer's problems defined in (1) is also dense.

Assuming that $F_{\epsilon,t}^b(S_{\epsilon,t}) < 1$[6] it follows that:

$$0 \geq \lim_{\epsilon \to 0}[u(\omega_0 + S_{\epsilon,t} - qY) + Eu(\omega_1 + Y)] - \lim_{\epsilon \to 0} V_{\epsilon}^s(t + \epsilon; \omega_i),$$

$$\lim_{\epsilon \to 0} V_{\epsilon}^s(t + \epsilon; \omega_0) \geq \lim_{\epsilon \to 0}[u(\omega_0 + S_{\epsilon,t} - qY) + Eu(\omega_1 + Y)]. \tag{9}$$

Since the seller is willing to trade at $S_{\epsilon,t}$ then:

$$\lim_{\epsilon \to 0}[u(\omega_0 + S_{\epsilon,t} - qY) + Eu(\omega_1 + Y)] \geq \lim_{\epsilon \to 0} V_{\epsilon}^s(t + \epsilon; \omega_0). \tag{10}$$

Therefore, from (9) and (10) obtain:

$$\lim_{\epsilon \to 0} V_{\epsilon}^s(t + \epsilon; \omega_0) = \lim_{\epsilon \to 0}[u(\omega_0 + S_{\epsilon,t} - qY) + Eu(\omega_1 + Y)]. \tag{11}$$

Define $\lim_{\epsilon \to 0} S_{\epsilon,t} \equiv S_t, \lim_{\epsilon \to 0} B_{\epsilon,t} \equiv B_t$[7] and $\lim_{\epsilon \to 0} \overline{V}_{\epsilon}^b(\omega_{i+1}) \equiv \overline{V}^b(\omega_{i+1})$. Also, define F_t^a, $a = b, s$ as the distributions of the limiting bids and ask prices, respectively. Equation (11) becomes:

$$V^s(t; \omega_0) = u(\omega_0 + S_t - qY) + Eu(\omega_1 + Y). \tag{12}$$

Since $V_{\epsilon}^s(t; \omega_0) \leq V_{\epsilon}^s(t'; \omega_0)$ for any two continuity points $t, t' \in (0, 1)$ such that $t > t'$ computing the limits for $\epsilon \to 0$ obtain:

$$u(\omega_0 + S_t - qY) + Eu(\omega_1 + Y) \leq u(\omega_0 + S_{t'} - qY) + Eu(\omega_1 + Y).$$

Since $u(\cdot)$ is monotonically increasing obtain:

$$S_t \leq S_{t'}, \tag{13}$$

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[6]This is equivalent to assuming that there is a positive mass of buyers who are willing to trade with some agent ω_0. A value of $F_{\epsilon,t}^b = 1$ for all ω's implies there is no trade. In section 2.4 we show that $F_{\epsilon,t}^a$ is degenerate only when the agents are risk neutral.

[7]The limits S_t and B_t exist and is unique since $u(.)$ is continuous and monotone.

for all continuity points $t, t' \in (t_i, t_{i+1})$ such that $t > t'$. A similar proof shows that:

$$B_t \geq B_{t'}. \tag{14}$$

The following theorem summarizes the result.

Theorem 1. *Suppose the trading sessions are frequent enough, i.e., ϵ sufficiently small. Then for all continuity points $t > t'$ a buyer with an endowment ω_0 has an optimizing bid such that:*

$$B_t(\omega_0) \geq B_{t'}(\omega_0),$$

and a seller with endowment ω_0 has an optimizing ask price such that:

$$S_t(\omega_0) \leq S_{t'}(\omega_0).$$

Letting $\lim_{\epsilon \to 0} S_{1-\epsilon} \equiv S_1$ the last session's limiting ask price S_1 can be solved by using equation (4) and (12) as:

$$u(\omega_0 - qY) + Eu(\omega_1 + \rho + Y) = u(\omega_0 + S_1) + Eu(\omega_1). \tag{15}$$

Similarly letting $\lim_{\epsilon \to 0} B_{1-\epsilon} \equiv B_1$, the limiting bid for the last session is given by:

$$u(\omega_0 - qY) + Eu(\omega_1 + Y) = u(\omega_0 - B_1 - qY) + Eu(\omega_1 + \rho + Y). \tag{16}$$

Let $v_{\epsilon,t}$ denote the expected volume of trade been defined as the proportion of exchanges taking place at a given session $t \in \mathcal{T}_\epsilon$, i.e.,

$$v_{\epsilon,t} = \int \int 1\{(\omega_0^b, \omega_0^s) : B_{\epsilon,t}(\omega_0^b) \geq S_{\epsilon,t}(\omega_0^s)\} dF_{\epsilon,t}^b(S_{\epsilon,t}(\omega_0^s)) dF_{\epsilon,t}^s(B_{\epsilon,t}(\omega_0^b)),$$

where $1\{\cdot\}$ is an indicator event and ω^a denotes the endowment of agent of type $a = b, s$.

Therefore the expected volume of trade is simply the probability of trade:

$$v_{\epsilon,t} = \Pr\{(\omega_0^b, \omega_0^s) : B_{\epsilon,t}(\omega_0^b) \geq S_{\epsilon,t}(\omega_0^s)\}.$$

Let now $v_t \equiv \lim_{\epsilon \to 0} v_{\epsilon,t}$ and $\lim_{\epsilon \to 0} v_{\epsilon,1-\epsilon} \equiv v$. The following theorem shows that the probability of trade is higher the closer to the date of the announcement. This implies that the volume of trade is non-decreasing. We shall further show in Section 2.4 and 2.5 that the probability of trade is either zero (when constant) or strictly increasing. Hence, $v_t = v_{t'}$ for $t > t'$ implies $v_t = 0$.

Theorem 2. *For any two continuity points* $t, t' \in (0,1)$ *such that* $t > t'$, $v \geq v_t \geq v_{t'}$.

Proof of Theorem 2: See Appendix.

Theorem 1 and 2 are intuitive: buyers start from a low bid and increase their offer as the deadline approaches. Sellers behave symmetrically. Though intuitive this might not necessarily be the case as the two opposite effects drive the dynamics of individual strategies. The first is due to the approaching deadline: given the distribution of the ask prices, buyers need to improve their offers so to increase the probability of trade as the number of future opportunities of trading deteriorates overtime; similarly for the sellers. The second effect comes from the dynamics of the price distributions of the counterparty: since the distribution of ask prices is improving overtime, each buyer should decrease (not increase) his bid. Similarly, each seller should increase (not decrease) her ask price. However, Equation (11) shows that the present distribution of the counterparty is irrelevant for determining the optimal ask price: as trading sessions become more frequent the value of waiting increases up to the point where the optimal asking price is the one at which the seller is just indifferent between consuming now the amount he is willing to trade for, or realizing the continuation value (*i.e.*, waiting for the next trading session); being the continuation value monotonic, so is the optimal sellers' strategy. The same holds for the buyers [8].

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[8]This reminds us of the optimal bidders' strategy in sealed-bid second price auctions where agents bid their "true value" of the object.

2.4 Risk neutral agents

In this section we study the patterns of trade in an economy entirely populated by risk neutral agents and show that in this case all transactions occur in the first trading session. This implies that if increasing trade is observed then agents must be risk averse and the loss of trade due to early arrival of information entails a loss of insurance opportunities.

Let agents' preferences be represented by any linear function $u(x) = k_1 + k_2 x$ where k_1 and k_2 are two constant (and k_2 is positive). Let the distribution of endowments as described above.

Lemma 2. *For all $t \in (0, 1)$ and for all $\epsilon > 0$ the continuation values are constant and they are given by:*

$$V_\epsilon^a(t; \omega_0) = V^a(t; \omega_0) = \overline{V}^a(\omega_0), \ \ a = b, s.$$

Proof of Lemma 2: See Appendix.

Theorem 3. *If agents are risk neutral then the bids and ask prices are endowment and ϵ-independent, constant and given by:*

$$B_{\epsilon,t} = B_1 = S_{\epsilon,t} = S_1 = E(\rho).$$

Proof of Theorem 3: See Appendix.

Theorem 3 states that risk neutral buyers and sellers have the same endowment-independent reservation price given by the expected value of the future returns. Since we assumed agents to trade when $b_t \geq s_t$, it follows that when agents are risk neutral they trade at the very first opportunity and the volume (*i.e.,* the probability) of trade is zero thereafter.

2.5 Strict risk aversion is necessary for the deadline effect

In the previous section we proved that when agents are risk neutral then bids and ask prices are stationary and degenerate for all trading sessions following the first one, *i.e.,* $t > \epsilon$. We now strengthen the result by ruling out such behavior for risk averse agents by showing that it can occurs if and only if agents are risk neutral. We start by showing that a constant

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volume of trade for all sessions after the first is possible if and only if the bid and ask prices are stationary. We then show that the latter occurs only under risk neutrality, which in turn implies by Theorem 3 that the volume of trade is nil for all $t > \epsilon$.

The following lemma shows that if trade is flat or there is no trade after the first trading session, then the distribution of bid and ask prices must be stationary.

Lemma 3. *The probability of trade is constant after the first trading session (i.e., $t > \epsilon$ for $t \in \mathcal{T}_\epsilon$) if and only if the bid and ask prices are stationary for almost all sellers and buyers, i.e., $v_t = v$ if and only if for some bid and ask prices $S_0(\omega)$ and $B_0(\omega)$:*

$$\Pr\{\omega_0 : S_t(\omega_0) = S(\omega_0) \text{ for all } t > \epsilon\} = 1, \tag{17}$$

and

$$\Pr\{\omega_0 : B_t(\omega_0) = B(\omega_0) \text{ for all } t > \epsilon\} = 1. \tag{18}$$

Proof of Lemma 3: See Appendix.

Note that (17) and (18) imply that $F_t^b(x) = F^b(x)$ and $F_t^s(x) = F^s(x)$ are stationary distributions. We are now in a position to prove the following result.

Theorem 4. *Buyers and sellers are risk neutral if and only if the probability of trade is 1.*

Proof of Theorem 4: See Appendix.

Theorem 4 implies that if trade is positive for $t > \epsilon$ and the schedule is credible, then its dynamics must be strictly increasing. In fact, it rules out the possibility that increasing dynamics might occur in an economy populated by risk neutral agents only.

2.6 The welfare effect of an early information release

Consider now an off schedule and unanticipated information release. Once information is conveyed to the markets, the asset looses its role of insurance instrument and becomes redundant. Its rate of return is now pinned down by the bond's rate of return as no buyer would offer more and no seller would ask less than the risk free rate. Being the price fixed, the OTC market becomes inactive after the session following the announcement.

It follows that an unanticipated early release of information entails a welfare loss equivalent to assigning to each active trader the utility obtained by holding the bond only.

This is obviously not the case if agents are risk neutral as agents are just indifferent between trading over-the-counter and in the bond market; moreover since in this case all trade occurs in the first trading session an early release of information has no effect on their welfare.

We can summarize the results in the following theorem.

Theorem 5. *An early release of information at any time t entails a welfare loss for any active risk averse trader $a = b, s$ with endowment ω equal to:*

$$V^a(t; \omega) - \overline{V}^a(\omega).$$

2.7. Stochastic deadlines

This section analyzes traders' behavior in the case of stochastic deadlines, *i.e.,* when traders assign a positive probability to the event the information provider acts at a $t < 1$.

In this section we show that such a probability is strictly positive at any time and the volume of trade is positive and uniform.

Suppose that the timing when the announcement is due is stochastic. Let there be an announcement at $t < 1$, then for every point $t \in (0, 1)$ define τ_t as:

$$1 - \tau_t = \Pr\left(\text{Announcement at } t\right). \tag{19}$$

Assumption 3. *The probability of an off schedule announcement is strictly positive for all t,* i.e.,

$$\sup_{t \in (0,1)} \tau_t < 1. \tag{20}$$

As before we consider each agents ϵ-step problem, where the agents failing to successfully exchange with the counterparty proceed to the next period. In this case, however, at each trading section following any t an announcement reaches the market with probability $1 - \tau_{t+\epsilon}$. In that event the agents that have not traded as yet, are left with the autarkic utility

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$\overline{V}^a_\epsilon(\omega_0), a = b, s.$

The optimization problem for any $\epsilon > 0$ can now be written as:

$$V^b_\epsilon(t, in; \omega_0, \tau_t) = \max_{b_t} \int_0^{b_t} [u(\omega_0 - \frac{b_t + x}{2} - qY_t) + Eu(\omega_1 + \rho + Y_t)]dF^s_{\epsilon,t}(x; \tau_t) \qquad (21)$$

$$+ (1 - F^s_{\epsilon,t}(b_t; \tau_t))[\tau_{t+\epsilon}V^b_\epsilon(t + \epsilon, in; \omega_0, \tau_t) + (1 - \tau_{t+\epsilon})u(\omega_1 + Y_t)],$$

$$Y_t \in \arg\max_{y_t} u(\omega - \frac{b_t + x}{2} - qy_t) + Eu(\omega_1 + \rho + y_t) \text{ for all } x < b_t$$

$$V^s_\epsilon(t, in; \omega_0, \tau_t) = \max_{s_t} \int_{s_t}^\infty [u(\omega_0 + \frac{s_t + x}{2} - qY_t) + Eu(\omega_1 + Y_t)]dF^b_{\epsilon,t}(x; \tau_t) \qquad (22)$$

$$+ F^b_{\epsilon,t}(s_t; \tau_t) [\tau_{t+\epsilon}V^s_\epsilon(t + \epsilon, in; \omega_0, \tau_t) + (1 - \tau_{t+\epsilon}) (\omega_0 + Y_t)],$$

$$Y_t \in \arg\max_{y_t} u(\omega + \frac{s_t + x}{2} - qy_t) + Eu(\omega_1 + y_t) \text{ for all } x < b_t.$$

Denote by $B_{\epsilon,t}(\omega_0, \tau_t)$ and $S_{\epsilon,t}(\omega_1, \tau_t)$ the solutions to problem (21) and (22), respectively. The next result shows that bid and ask prices as well as bond holdings, are independent of time and frequency (but not of the endowment). This follows from the stationarity of the problem under stochastic deadlines.

Theorem 6. *Under Assumption (3), for a given ω_0 and for any trading frequency $\varepsilon > 0$:*

a) the value function $V^a_\epsilon(t, in; \omega_0, \tau_t) = V^a(t, \omega_0) = \overline{V}^a(\omega_0), a = b, s;$

b) the bids and ask prices, i.e. *$B_{\epsilon,t}(\omega_0, \tau_t)$ and $S_{\epsilon,t}(\omega_0, \tau_t)$, are stationary and τ_t-independent.*

Proof of Theorem 6: See Appendix.

The following theorem shows that risk averse agents facing an uncertain deadline behave at each instant as they would behave at the last trading opportunity in a credible schedule regime, *i.e.*, they use their last period's bids and ask prices. Recall that these are the prices that give the the same level of utility of the autarkic equilibrium. It follows that the trading volumes remains constant.

Theorem 7. *Under stochastic deadlines the bids, ask prices and the volume of trade are given by:*

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a) $S_{\epsilon,t}(\omega_0, \tau_t) = S_1(\omega), B_{\epsilon,t}(\omega_0, \tau_t) = B_1(\omega),$

b) $v_\tau = \Pr\{\omega_0 : B_{\epsilon,t}(\omega_0, \tau_t) \geq S_{\epsilon,t}(\omega_0, \tau_t)\} = v_1,$

where B_1, S_1 are the last session prices (see (15) and (16)) and v_1 is the last session volume under a known deadline.

Proof of Theorem 7: See Appendix.

From Theorem 7 it is possible to conclude the theoretical part with the following result comparing the volume of trade under alternative credibility of the monetary policy schedule:

Corollary 1. *The volume of trade is higher under stochastic deadlines than under credible schedule announcements.*

Welfare effect: Notice that since the reservation value for any given agent is constant and set equal to the value of autarky, there are no adverse effects of an early release of information and hence there is no welfare effect. However, in the scheduled case the expected utility of a trader at $t < 1$ is higher than in the case of unscheduled announcements.

3. The Empirical Model

Most independent central banks, including the U.S. Fed, the Bank of England and the ECB, deliver their monetary policy decisions to the public by announcing interest rate levels at scheduled and publicly available dates. Scheduling monetary policy, it is usually argued, increases "transparency, accountability and the dialogue with the public," (Bank of Canada, (2000)). Monetary policy authorities retain the ability to act off schedule, though this might undermine their policy's credibility. When the schedule is credible, off schedule announcements are often said to "surprise" the markets. Other monetary authorities, *e.g.,* the Reserve Bank of India, prefer to exercise discretion by informing the markets about rate changes whenever considered appropriate.

Under the hypotheses of rational expectations, no transaction costs and complete competitive markets, the procedures and timing of the announcements would hardly matter as prices would perfectly reflect information and traders would continuously adjust their portfolios.

Our theoretical model argues that this might not be the case in over-the-counter markets, by showing that fixing the dates of information delivery changes agents' optimal trading strategies and in turn the dynamics of trade. This occurs only when the schedule is credible. Moreover, surprising the market by moving forward the information release might prevent risk-sharing improving trade to take place.

In this section we apply these theoretical findings to the FOMC monetary policy scheduling from 3rd January 1995 to 31th July 2010 and look at the impact on the dynamics of trade of the CBOT 30-Day Federal Funds Futures market. These are contracts of $5mil size on the daily federal funds overnight rate reported by the New York Fed[9]. The 1995 date started a period lasting till 1998 during which the monetary committee was very consistent in following the schedule. Throughout this 15 years' period the FOMC had 128 scheduled meetings and 17 off schedule meetings[10]. Only 4 of these latter led to a change of rate: the quarter point reduction on 18/19 of October 1998 and the half point cuts on 3rd January 2001, 18th April and 17th September 2001.

Our first aim is to identify the periods of credible monetary policy schedule: the model shows that these are characterized by a significant deadline effect prior to the scheduled announcements. Recall that a deadline effects occur only under credible scheduling (Theorem 1 and 7) and if and only if agents are risk averse (Theorem 2 and 3). In order to identify the periods of credible monetary policy scheduling we use rolling windows of 400 days with a 60 days overlapping gap. Our second aim is to quantify the loss of trade due to unanticipated information releases. For the latter we start by looking at meetings occurring during periods of credible scheduling but off the regular schedule. The absence of a deadline effect prior to these events would indicate that these were unanticipated. The implication is that off schedule announcements must have conveyed information to the active risk averse agents prior to risk sharing trading, amounting to a welfare loss. We proxy the latter by looking at the average of lost trade due to an early information release.

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[9]See also http://www.cbot.com/cbot/pub/cont_detail/0,3206,1525+14446,00.html.

[10]When the off schedule meetings occur at non-trading days, we take the first trading day after the meeting as the effective announcement day.

3.1 Modeling trading volume

We look at the trading volume of short term interest rate futures as a function of changes of the federal funds rate following the announcements of scheduled and off schedule meetings by analyzing the following model:

$$v_t = f(r_{t_i} - E_t(r_{t_{i+1}})) \ + \sum_{j=-J}^{J}(\alpha_j^0 SD_{t-j}^0 + \beta_j UD_{t-j}^+ + \alpha_j SD_{t-j}^+) + \epsilon_t,$$

where v_t is the daily volume traded of CBOT® 30-Day Federal Funds Futures at time t in the Chicago Board of Trade. r_t is the federal funds rate at t determined by the FOMC after the meetings, either scheduled or unscheduled. The volume of trading v_t depends on the gap $r_{t_i} - E_t(r_{t_{i+1}})$, where r_{t_i} is the actual interest rate at t before the meeting at $i+1$ and $E_t(r_{t_{i+1}})$ is the market expectation of change of rate at the $i+1^{th}$ meeting given public information at time t.

In order to capture the effects on the volume of trade when the interest rate changes, we differentiate between the announcements that lead to a change the federal fund rate from the ones that do not by introducing the following dummy variables:

$$SD_t^0 = I\left[|\Delta r_t| = 0 \text{ and there is a scheduled meeting at } t\right],$$

and

$$SD_t^+ = I\left[|\Delta r_t| \neq 0 \text{ and there is a scheduled meeting at } t\right],$$

where I is an indicator function. SD_{t-j}^0 and SD_{t-j}^+, $j = 1, .., J$ capture the effects of possible excess trading the day before scheduled announcements whereas to capture the increase in trade the day after the announcements we include the dummies SD_{t+j}^0 and SD_{t+j}^+, $j = 1, .., J$. We also introduce a separate but similar set of variables in order to capture the effects of the surprise, or off schedule rate changes, by introducing the following dummy variable:

$$UD_t^+ = I\left[|\Delta r_t| \neq 0 \text{ and there is a off schedule meeting at } t\right].$$

In order to capture the effects of possible excess trading the day before the off schedule announcement we use $UD^+_{t-j}, j = 1, .., J$. For the day after announcement effects we include the lead dummies $UD^+_{t+j}, j = 1, .., J$.

We model the difference in interest rate expectations from the realized value $r_t - E_t(r_i)$ as a function of the difference in output growth i.e., $\Delta y_t = y_{t+1} - E_{t+1}(y_t)$, and in inflation expectation[11] $\Delta \pi_t = \pi_{t+1} - E_{t+1}(\pi_t)$. We model volume as a function of absolute magnitude changes of the difference between the median and the forecast values of output growth and inflation. We obtain:

$$
\begin{aligned}
v_t = \ & c + \gamma_1 |\Delta \pi_t| + \gamma_2 |\Delta y_t| \\
& + \sum_{j=-J}^{J} (\alpha_j^0 SD_{t-j}^0 + \beta_j UD_{t-j}^+ + \alpha_j SD_{t-j}^+) + \epsilon_t.
\end{aligned}
\tag{23}
$$

3.2 Results

Our test identifies two sub-periods in our data set: the first, till September 2001, where there is statistically significant excess trade (i.e., a deadline effect) two and one day before a announcement of a rate change. The second period following September 2001 where there is no significant change in excess trade prior to scheduled announcements. Figure 1 plots the t-values of α_1^0 and α_2^0 for the excess trade one day before and two days before a scheduled announcement for the period January 1995-September 2001 and October 2010-July 2010, respectively. Each bar on the graphs represents a 400-day window. If in any of the rolling periods one of the three events SD_{t-j}^+, SD^0 or UD^+ does no occur we drop the respective dummy variables for that period and the corresponding a window in the figure will appear blank. The three horizontal lines indicate the significance at 1%, 5% and 10% levels. Figure 2 represent the plots for the two periods for the average volume of trade the day of a scheduled meetings followed by an interest rate change, one and two days prior to that meeting along

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[11]The data on expectations are obtained from Datastream.

with the average of all the other days.

[INSERT FIGURE 1]

[INSERT FIGURE 2]

An obvious question to ask is why the credibility of monetary policy has been affected after September 2001. Although several reasons can be attributed to changes in traders' beliefs, our model is silent about the reasons affecting the schedule's credibility.

We can then turn to the to evaluation of the loss of trading volume due to unanticipated information arrivals. Recall that the emphasis is on the *timing* of information arrival and not on its content. The relevant events in this case are the four off schedule announcements that led to a change in interest rate. Notice that all of them occurred before October 2001, the period with significant deadline effects of schedule announcements and hence of credible scheduling. We first show that these were indeed surprises to the market by verifying the absence of a deadline effect the days prior to these announcements. Table 1 summarizes the results of the tests[12].

[INSERT TABLE 1 AND TABLE 2 HERE]

Finally, in order to quantify the loss of trade we look at the average excess trade one and two days before each off schedule announcement compared to the value of the intercept in the table, *i.e.*, $\left(\widehat{\alpha}_j - \widehat{\beta}_j\right)/\widehat{c}$, $j = -1, -2$. We find that an average excess trade of 49% the day before and 37% two days before the announcement.

4. Conclusion

In this paper we argue that scheduling the communication of payoff relevant public information changes financial markets' behavior non-trivially by entailing a deadline effect. The

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[12]Since in the second period there are no off schedule announcements followed by changes in interest rates we drop the variables UD_{t-j}^+ during that period.

theoretical contribution has shown that observables like trading volume dynamics can iden-
tify welfare effects of credible scheduling. We apply the theoretical model to the FOMC
monetary policy announcements. We first identify the periods of credible monetary policy
and then we show that in those periods unscheduled announcements entail a loss of trade.
Some final observations are in order: 1) our welfare analysis focuses on the general equilib-
rium effect leading to a loss of insurance opportunities and abstracts from other potentially
beneficial effects that might arise from an early release of information. As pointed out by
Gottardi and Rahi (2008), if there is room for trade after the new information has reached
the market, agents can achieve a larger set of state contingent payoffs by conditioning their
portfolios on this information: if markets are sufficiently incomplete, the latter positive effect
might overcome the welfare loss due to the Hirshleifer effect. This important point is beyond
the aim of our analysis; 2) our paper is also silent about trade increases observed after a
FOMC announcement. We do not account for this effect though this increase clearly shows
up in our data. It is well known that trade for many asset classes increases right after news
are released. Following the argument provided in our model, one may conjecture that in
the presence of agents with different degrees of risk aversion, risk neutral agents trade first
(right after the news reaches the market) and risk averse agents only later; 3) the dynamics
of trade in interest rates futures is substantially different than the one observed prior to
scheduled corporate announcements where trade is depressed rather than increased. The
financial economics literature has identified informational asymmetries as the main reason
for the volume of trade to decrease as uninformed agents avoid the exchange with informed
counterparties[13]. If trading volume before scheduled announcements is indeed correlated
with the extent of information asymmetries then our empirical findings would imply that
there are little informational asymmetries on monetary policy decisions.

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[13]For the theoretical literature see Admati and Pfeinderer (1988) and Forster and Viswanathan (1990).
For the empirical studies see Chae (2005). For alternative explanations see George, Kaul, and Nimalendran
(1994)).

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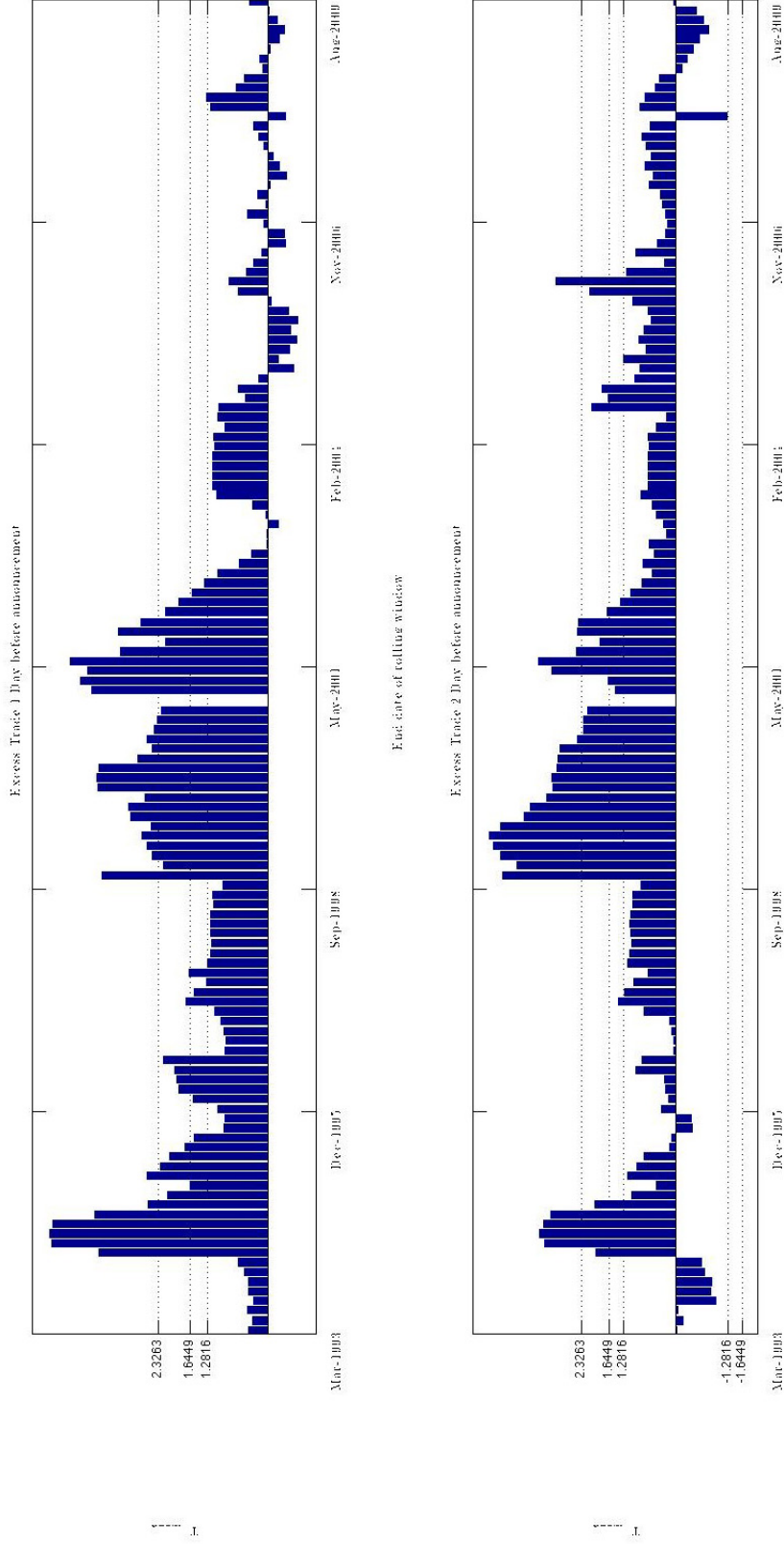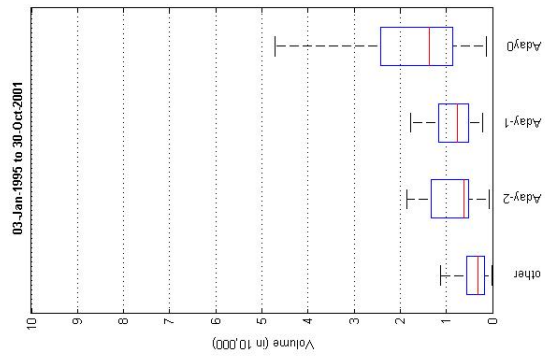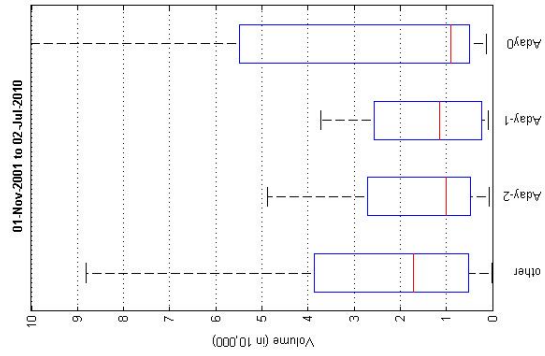
Figure 1: Rolling window estimation

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Figure 2: Excess Trade Before and After October-2001

Table 1: Estimation Results. Dependent Variable: Volume

From : 03-Jan-1995 to 30-Sept-2001			
$\overline{R}^2$	0.16		
DW	0.66		
T	1673	K	18
Variable	Coefficient	t-statistic	t-probability
const	4201.94	33.30	0.00
Δy_t	95.45	0.10	0.92
$\Delta \pi_t$	4246.67	0.86	0.39
SD_{-2}^+	4985.58	4.48	0.00
SD_{-1}^+	5911.39	5.32	0.00
SD_0^+	13514.13	12.14	0.00
SD_1^+	8618.00	7.75	0.00
SD_2^+	4369.75	3.93	0.00
SD_{-2}^0	-810.41	-1.01	0.31
SD_{-1}^0	-610.00	-0.76	0.45
SD_0^0	1758.77	2.19	0.03
SD_1^0	-33.51	-0.04	0.97
SD_2^0	-1140.34	-1.42	0.16
UD_{-2}^+	3444.39	1.27	0.20
UD_{-1}^+	3865.61	1.42	0.15
UD_0^+	17781.72	6.56	0.00
UD_1^+	13043.72	4.81	0.00
UD_2^+	5596.06	2.07	0.04

Table 2: Estimation Results. Dependent Variable: Volume

From : 01-Oct-2001 to 02-Jul-2010			
$\overline{R}^2$	0.02		
DW	0.37		
T	2176	K	13
Variable	Coefficient	t-statistic	t-probability
const	25837.46	35.76	0.00
Δy_t	4259.44	0.86	0.39
$\Delta \pi_t$	-4185.36	-0.35	0.73
SD_{-2}^+	-2913.41	-0.50	0.61
SD_{-1}^+	-4773.36	-0.83	0.41
SD_0^+	14986.61	2.59	0.01
SD_1^+	3983.38	0.69	0.49
SD_2^+	-5517.07	-0.96	0.34
SD_{-2}^0	8067.29	1.66	0.10
SD_{-1}^0	12530.17	2.57	0.01
SD_0^0	24585.18	5.04	0.00
SD_1^0	11644.65	2.39	0.02
SD_2^0	7321.35	1.49	0.14

Appendix

Definition 1. *By equilibrium of $\mathcal{G}_\epsilon$ we mean the* subgame perfect equilibrium, *i.e.: for each buyer at $t \in ((l-1)\epsilon, l\epsilon]$ with history $h_t = in$ and endowment ω_0^b the equilibrium strategy $(Y_{\epsilon,t}^*(\omega_0^b), B_{\epsilon,t}^*(\omega_0^b))$ is the solution to problem (1) when $\{(Y_{\epsilon,t}^*(\omega_0^b), B_{\epsilon,t}^*(\omega_0^b)) : t \in (l\epsilon, L\epsilon]\}$ are his future strategies and $\{F_{\epsilon,t}^{s*} : t \in ((l-1)\epsilon, L\epsilon]\}$ are the present and future distributions of the active sellers equilibrium asking prices $\{S_{\epsilon,t}^*(\omega_0^s)) : t \in ((l-1)\epsilon, L\epsilon]\}$ for all ω_0^s; similarly, for each seller at $t \in ((l-1)\epsilon, l\epsilon]$ with history $h_t = in$ and endowment ω_0^s the equilibrium strategy $(Y_{\epsilon,t}^*(\omega_0^s), S_{\epsilon,t}^*(\omega_0^s))$ is the solution to the problem (2) when $\{(Y_{\epsilon,t}^*(\omega_0^s), S_{\epsilon,t}^*(\omega_0^s)) : t \in (l\epsilon, L\epsilon]\}$ are her future strategies and $\{F_{\epsilon,t}^{b*} : t \in (l\epsilon, L\epsilon]\}$ are the present and future distributions of active buyers equilibrium bids $\{B_{\epsilon,t}^*(\omega_0^b) : t \in (l\epsilon, L\epsilon]\}$ for all ω_0^s. Finally, the bond market clears, i.e. at equilibrium q_ϵ^* is such that:*

$$\int_0^1 \int \int [Y_{\epsilon,t}^*(\omega_0^b) + Y_{\epsilon,t}^*(\omega_0^s)] dF_{\epsilon,t}^{b*}(S_{\epsilon,t}^*(\omega_0^s)) dF_{\epsilon,t}^{s*}(B_{\epsilon,t}^*(\omega_0^b)) dt = 0$$

In Lemma 1 we show that the optimal bond's holding is independent of ϵ and t. The existence of the equilibrium can be shown by backward induction and by noting that the only relevant state is in. The equilibrium in session $t = L\epsilon$ follows from Mas-Colell (1984) and hence for all trading session $t \in \{\epsilon, \dots, (L-1)\epsilon\}$.

Proof of Lemma 1:

Define the following static problem:

$$G(s, Y : \omega, V, F^b) = \int_s^\infty [u(\omega + \frac{s+x}{2} - qY) + Eu(\omega_1 + Y)] dF^b(x) + F^b(s)V,$$

$$s.t. \ Y \in \arg\max_y u(\omega + \frac{s+x}{2} - qy) + Eu(\omega_1 + y) \text{for all } x > s_t$$

or:

$$G(s, Y : \omega, V, F^b) = \int_s^\infty [u(\omega + \frac{s+x}{2} - qY) + Eu(\omega_1 - Y)]dF^b(x) + F^b(s)V,$$

$$\text{s.t } u'\left(\omega + \frac{s+x}{2} - qY\right) = Eu'(\omega_1 + Y)$$

Writing the Lagrangian as:

$$\mathcal{L}(s, x, y : \ \omega, V, F^b) = [u(\omega + \tfrac{s+x}{2} - qy) + Eu(\omega_1 + y)]I\,[x > s] + I\,[x < s]\,V$$
$$+ \lambda\,(x)\left[u'\left(\omega + \tfrac{s+x}{2} - qy\right) - Eu'\,(\omega_1 + y)\right],$$

we note that the Lagrangian is independent of F^b.

Letting Y the argmax of the Lagrangian, we obtain:

$$\frac{\partial Y}{\partial V} = -\left.\frac{\partial \mathcal{L}}{\partial V \partial y}\middle/\frac{\partial \mathcal{L}}{\partial^2 y}\right|_{y=Y}.$$

Computing the derivative with respect to y :

$$\frac{\partial \mathcal{L}(s, x, y : \omega, V, F^b)}{\partial y} = u'(\omega + \frac{s+x}{2} - qy) - Eu'(\omega_1 + \rho + y)] +$$
$$\lambda\,(x)\left[u''\left(\omega + \frac{s+x}{2} - qy\right) + Eu''\,(\omega_1 + \rho + y)\right]$$
$$= \lambda\,(x)\left[u''\left(\omega + \frac{s+x}{2} - qy\right) + Eu''\,(\omega_1 + \rho + y)\right]$$

$$\frac{\partial \mathcal{L}(s, y : \omega, V, F^b)}{\partial y \partial V} = 0,$$

therefore obtain:

$$\frac{\partial Y}{\partial V} = 0. \tag{24}$$

hence Y does not change with V.

Note that

$$V_\epsilon^s(t; \omega_0) = \max_s G(s, Y : \omega_0, V_\epsilon^s(t + \epsilon; \omega_0), F_t^b)$$

$$V_\epsilon^s(t'; \omega_0) = \max_s G(s, Y : \omega_0, V_\epsilon^s(t'; \omega_0), F_{t'}^b) \text{ for } t > t'.$$

Since by (24) $Y_t = \arg\max_y \mathcal{L}(s, x, y : \omega, V_\epsilon^s(t+\epsilon; \omega_0), F_t^b)$ does not change with $V_\epsilon^s(t+\epsilon; \omega_0)$, we have:

$$Y_t = Y_{t'} = Y.$$

The ϵ-independence of the last period value functions follow from equations (3) and (4).

$\square$

The following two results (Lemma A1 and Lemma A2) will be useful in proving Lemma2 in the text.

Lemma A 1. *The risk neutral agent's value function is such that:*

$$\frac{\partial V_\epsilon^a(t; \omega_0)}{\partial \omega_0} = k_2 \text{ for all } t \in \mathcal{T}_\epsilon, \ a = b, s. \tag{25}$$

Proof of Lemma A 1: Consider the ϵ-step problem for the trader of type s and endowment ω_0. For $t = 1$ the derivative of the last trading session value function (4) is given by:

$$\frac{\partial V_\epsilon^s(1; \omega_0)}{\partial \omega_0} = \frac{\partial u(\omega_0 - qY) + Eu(\omega_1 + \rho + Y)}{\partial \omega_0} = k_2.$$

If (25) holds for $t + \epsilon$ then it holds for any $t \in \mathcal{T}_\epsilon$. In fact since $S_{\epsilon,t}$ is the argmax of the problem in (2) it follows that:

$$\frac{\partial V_\epsilon^s(t; \omega_0)}{\partial \omega_0} = \int_{S_{\epsilon,t}}^{\infty} \frac{\partial}{\partial \omega_0} [u(\omega_0 + \frac{S_{\epsilon,t} + x}{2} - qY) + Eu(\omega_1 + Y)] dF_{\epsilon,t}^b(x)$$

$$+ F_{\epsilon,t}^b(S_{\epsilon,t}) \frac{\partial}{\partial \omega_0} V_\epsilon^s(t + \epsilon; \omega_0).$$

Therefore:

$$\frac{\partial V_\epsilon^s(t;\omega_0)}{\partial\omega_0} = \int_{S_{\epsilon,t}}^{\infty} k_2 dF_{\epsilon,t}^b(x) + F_{\epsilon,t}^b(S_{\epsilon,t})k_2 = k_2.$$

It now suffices to notice that this is true for all ϵ-step problems to obtain the result. The proof is similar for buyers. $\qquad\square$

Lemma A 2. *The risk neutral agents' reservation price is endowment independent, i.e.:*

$$\frac{\partial B_{\epsilon,t}}{\partial\omega_0} = \frac{\partial S_{\epsilon,t}}{\partial\omega_0} = 0, \text{ for all } t \in \mathcal{T}_\epsilon.$$

Proof of Lemma A 2: Consider a risk-neutral agent as in the proof of Lemma A 1. For a given $t \in \mathcal{T}_\epsilon$ define:

$$G_{\epsilon,t}(s_t,\omega_0) = \int_{s_t}^{\infty} [u(\omega_0 + \frac{s_t+x}{2} - qY) + Eu(\omega_1 + Y)]dF_{\epsilon,t}^b(x)$$
$$+ F_{\epsilon,t}^b(s_t)V_\epsilon^s(t+\epsilon;\omega_0),$$

and notice that:

$$G_{\epsilon,t}(S_{\epsilon,t},\omega_0) = V_\epsilon^s(t;\omega_0).$$

Being $S_{\epsilon,t}$ the argmax and the second derivative of $G_{\epsilon,t}$ negative, we obtain :

$$\frac{\partial S_{\epsilon,t}}{\partial\omega_0} = -\left.\frac{\frac{\partial G_{\epsilon,t}(s_t,\omega_0)}{\partial s_t \partial\omega_0}}{\frac{\partial G_{\epsilon,t}(s_t,\omega_0)}{\partial s_t \partial s_t}}\right|_{s_t=S_{\epsilon,t}}.$$

Computing the derivative with respect to ω_0 and using Lemma 1 obtain:

$$\frac{\partial G_{\epsilon,t}(s_t,\omega_0)}{\partial\omega_0} = \int_{s_t}^{\infty} \frac{\partial}{\partial\omega_0}[u(\omega_0 + \frac{s_t+x}{2} - qY) + Eu(\omega_1+)]dF_{\epsilon,t}^b(x)$$
$$+ F_{\epsilon,t}^b(s_t)\frac{\partial}{\partial\omega_0}V_\epsilon^s(t+\epsilon;\omega_0)$$
$$= \int_{s_t}^{\infty} k_2 dF_{\epsilon,t}^b(x) + F_{\epsilon,t}^b(s_t)k_2 = k_2,$$

then:
$$\frac{\partial G_{\epsilon,t}(s_t, \omega_0)}{\partial s_t \partial \omega_0} = 0.$$

It follows that:
$$\frac{\partial S_{\epsilon,t}}{\partial \omega_0} = 0, \text{ for all } t \in \mathcal{T}_\epsilon.$$

The proof is similar for the buyers. □

Proof of Theorem 2: Consider two continuity points $t, t' \in (0,1)$ such that $t > t'$. Consider also a seller s with endowment ω_0^s and a buyer b with endowment ω_0^b such that for some "trading frequency" $\epsilon > 0$, $B_{\epsilon,t'}(\omega_0^b) > S_{\epsilon,t'}(\omega_0^s)$. Then from Proposition (13) and (14) we have $B_{\epsilon,t}(\omega_0^b) \geq B_{\epsilon,t'}(\omega_0^s)$ and $S_{\epsilon,t}(\omega_0^s) \leq S_{\epsilon,t'}(\omega_0^s)$, we obtain $B_{\epsilon,t}(\omega_0^b) > S_{\epsilon,t}(\omega_0^s)$.

Therefore,

$$\{(\omega_0^b, \omega_0^s) : B_{\epsilon,t}(\omega_0^b) > S_{\epsilon,t}(\omega_0^s)\} \supseteq \{(\omega_0^b, \omega_0^s) : B_{\epsilon,t'}(\omega_0^b) > S_{\epsilon,t'}(\omega_0^s)\}.$$

It follows that:

$$v_{\epsilon,t} = \Pr\{(\omega_0^b, \omega_0^s) : B_{\epsilon,t}(\omega_0^b) > S_{\epsilon,t}(\omega_0^s)\}$$
$$\geq \Pr\{(\omega_0^b, \omega_0^s) : B_{\epsilon,t'}(\omega_0^b) > S_{\epsilon,t'}(\omega_0^s)\} = v_{\epsilon,t'}.$$

□

Proof of Lemma 2: Consider the ϵ-step problem. By Lemma A 2 since the ask prices $S_{\epsilon,t}$ and the bid prices $B_{\epsilon,t}$ are independent of the endowment ω_0 at each $t \in \mathcal{T}_\epsilon$ then the distribution $F_{t,\epsilon}^b(x)$ is degenerate. Let $\bar{B}_{\epsilon,t}$ be the degenerate bid of the buyers, then:

$$dF_{\epsilon,t}^b(x) \quad = 1 \text{ if } x = \bar{B}_{t,\epsilon}$$
$$= 0 \text{ otherwise.}$$

Notice that since the distribution of bids is degenerate at $\bar{B}_{\epsilon,t}$, no seller will ask strictly less than $\bar{B}_{\epsilon,t}$ and hence it will be optimal to proceed to the next interval. Therefore $F_{\epsilon,t}^b(S_{\epsilon,t}) = 1$.

Hence it follows from (5) that:

$$V_\epsilon^s(t; \omega_0) = V_\epsilon^s(t + \epsilon; \omega_0) \text{ for all } t \in \mathcal{T}_\epsilon \text{ and for all } \epsilon > 0.$$

Since $V_\epsilon^s(1; \omega_0) = u(\omega_0 - qY) + Eu(\omega_1 + \rho + Y)$ from equation (4) the result follows. The proof is similar for the buyers' reservation price. $\qquad\square$

Proof of Theorem 3: Consider two points $t, t' \in (0, 1)$ such that $t > t'$. By Lemma 2 and computing the limit as $\epsilon \to 0$:

$$V^s(t; \omega_0) = V^s(t'; \omega_0)$$

$$u(\omega_0 + S_t - qY) + Eu(\omega_1 + Y) = u(\omega_0 + S_{t'} - qY) + Eu(\omega_1 + Y).$$

It follows that:

$$S_t = S_{t'}.$$

The same is true for the bid prices B_t.

In the last trading session, the ask price S_1 can be solved as:

$$u(\omega_0 + S_1 - qY) + Eu(\omega_1 + Y) = u(\omega_0 - qY) + Eu(\omega_1 + \rho + Y),$$

or

$$S_1 = \frac{E(\omega_1 + \rho + Y) - E(\omega_1 + Y)}{k_2} \text{ for all } \omega_0.$$

Similarly, in the last period the bid B_0 can be solved as:

$$u(\omega_0 - B_1 - qY) + Eu(\omega_1 + \rho + Y) = u(\omega_0 - qY) + Eu(\omega_1 + Y).$$

It follows that:

$$B_1^* = \frac{Eu(\omega_1 + \rho + Y) - Eu(\omega_1 + Y)}{k_2} \text{ for all } \omega_0.$$

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Then obtain:
$$P_1 = B_1 = S_1 = \frac{Eu(\omega_1 + \rho + Y) - Eu(\omega_1 + Y)}{k_2} = \rho. \qquad (26)$$

$\square$

Proof of Lemma 3: Suppose equations (17) and (18) hold. Then for any pair $t \neq t' \in (0,1)$ following the first trading session and a trading frequency $\epsilon > 0$:

$$v_t = \Pr\left\{(\omega_0^b, \omega_0^s) : B(\omega_0^b) \geq S(\omega_0^s)\right\} = v_{t'}.$$

If instead (17) does not hold then by (13) there exists a $\underline{t}$ such that for all $t > \underline{t}$ there exists a subset of sellers such that:

$$\Omega_{\underline{t}} = \{\omega_0^s : S_{\underline{t}}(\omega_0^s) > S_t(\omega_0^s)\} \text{ and } \Pr(\Omega_{\underline{t}}) > 0.$$

This implies:

$$v_{\underline{t}} = \Pr\left\{(\omega_0^b, \omega_0^s) : B(\omega_0^b) \geq S_{\underline{t}}(\omega_0^s)\right\}$$
$$= \Pr\left\{(\omega_0^b, \omega_0^s) : B(\omega_0^b) \geq S_{\underline{t}}(\omega_0^s) \text{ s.t. } \omega_0^s \in \Omega_{\underline{t}}\right\}$$
$$+ \Pr\left\{(\omega_0^b, \omega_0^s) : B(\omega_0^b) \geq S(\omega_0^s) \text{ s.t. } \omega_0^s \in \Omega_{\underline{t}}^c\right\}$$
$$< \Pr\left\{(\omega_0^b, \omega_0^s) : B(\omega_0^b) \geq S_t(\omega_0^s) \text{ s.t. } \omega_0^s \in \Omega_{\underline{t}})\right\}$$
$$+ \Pr\left\{(\omega_0^b, \omega_0^s) : B(\omega_0^b) \geq S(\omega_0^s) \text{ s.t. } \omega_0^s \in \Omega_{\underline{t}}^c\right\}$$
$$= \Pr\left\{(\omega_0^b, \omega_0^s) : B(\omega_0^b) \geq S_t(\omega_0^s)\right\} = v_t.$$

a contradiction. Similarly if (18) does not hold there is a contradiction. $\square$

Proof of Theorem 4: Consider a seller with endowment ω_0. Since:

$$u(\omega_0 + S_1 - qY) + Eu(\omega_1 + Y) = u(\omega_0 - qY) + Eu(\omega_1 + \rho + Y),$$

and $Eu(\omega_1 + Y) < Eu(\omega_1 + \rho + Y)$ then $S_1 > 0$. By Lemma 3 and the envelope theorem:

$$\frac{\partial V^s(t; \omega_0)}{\partial \omega_0} = \int_{S_1}^{\infty} \frac{\partial}{\partial \omega_0} \left[u(\omega_0 + \frac{S_1 + x}{2} - qY) + Eu(\omega_1 + Y) \right] dF^b(x),$$

$$+ F^b(S_1) \frac{\partial}{\partial \omega_0} V^s(t + \epsilon; \omega_0),$$

$$u'(\omega_0 - qY) = \int_{S_1}^{\infty} u'(\omega_0 + \frac{S_1 + x}{2} - qY) dF^b(x) + F^b(S_1) u'(\omega_0 - qY)$$

$$0 = \int_{S_1}^{\infty} \left[u'(\omega_0 - qY) - u'(\omega_0 + \frac{S_1 + x}{2} - qY) \right] dF^b(x) \text{ for all } \omega_0 \text{ and } x \geq S_1.$$

Since $u'' \leq 0$ we have $u'(\omega_0 + \rho + Y) - u'(\omega_0 + \frac{S_1 + x}{2} + Y) \geq 0$ for almost all $x \geq S_1$. So the last equation is true if:

$$u'(\omega_0 - qY) = u'(\omega_i + \frac{S_1 + x}{2} - qY) \text{ for a.e. } x \geq S_1, \text{ or} \tag{27}$$

$$dF^b(x) = 0 \text{ for all } x \geq S_1. \tag{28}$$

If (27) is true then the utility is linear. If (28) is true then the distribution of bid prices are degenerate at some B' for all ω_0's:

$$u(\omega_0 - B' - qY) + Eu(\omega_1 + \rho + Y) = u(\omega_0 - qY) + Eu(\omega_1 + Y), \text{ for all } \omega_0.$$

Computing the derivatives obtain:

$$u'(\omega_0 - B' - qY) = u'(\omega_0 - qY), \text{ for all } \omega_0,$$

implying that the utility function is linear. $\qquad \square$

Proof of Theorem 6: Consider a seller with endowment ω_0. Consider the lagrangian

function:

$$\mathcal{L}(s,x,y \ : \ \omega, V, \overline{V}) = [u(\omega + \frac{s+x}{2} + y) + Eu(\omega_1 + \rho - y)]I[x > s] + I[x < s][\tau V + (1-\tau)\overline{V}]$$
$$+ \lambda(x)\left[u'\left(\omega + \frac{s+x}{2} + y\right) - Eu'(\omega_1 + \rho - y)\right].$$

Notice that $V_\epsilon^s(t; \omega_0, \tau_t) = \int_{S_{\epsilon,t}(\omega_0, \tau_t)}^\infty \mathcal{L}(s, x, Y_{\varepsilon,t} \ : \ \omega, V_\epsilon^s(t + \epsilon; \omega_0, \tau_{t+\epsilon}), \overline{V}_\epsilon^s(\omega_1))dF_{\epsilon,t}^b(x; \tau_t)$. Then define a functional $\Phi : \mathcal{V} \to \mathcal{V}$, where $\mathcal{V}$ is the space of bounded continuous functions such that:

$$\Phi\left(V_\epsilon^s(t; \omega_0, \tau_t)\right) = \int_{S_{\epsilon,t}(\omega_0, \tau_t)}^\infty \mathcal{L}(S_{\epsilon,t}(\omega_0, \tau_t), x, Y_{\varepsilon,t} : \omega, V_\epsilon^s(t; \omega_0, \tau_t), \overline{V}_\epsilon^s(\omega_1))dF_{\epsilon,t}^b(x; \tau_t).$$

where $(S_{\epsilon,t}(\omega_0, \tau_t), Y_{\varepsilon,t})$ is the argmax of (22). We show that Φ is a contraction mapping. Let $V_\epsilon^s(t; \omega_0, \tau_t)$ and $V_\epsilon'^s(t; \omega_0, \tau_t)$ be two functions then:

$$\Phi(V_\epsilon^s(t; \omega_0, \tau_t)) - \Phi(V_\epsilon'^s(t; \omega_0, \tau_t)) = \tau_{t+\epsilon}F_{\epsilon,t}^b(S_{\epsilon,t}-)(V_\epsilon^s(t + \epsilon; \omega_0, \tau_t) - V_\epsilon'^s(t + \epsilon; \omega_0, \tau_t)).$$

Since $\sup_{t \in (0,\infty)} \tau_t < 1$ and $F_{\epsilon,t}^b(S_{\epsilon,t}(\omega_0, \tau_t)) \le 1$, we can choose a $\delta < 1$ such that $\sup_{t \in (0,\infty)} \tau_t F_{\epsilon,t}^b(S_{\epsilon,t}(\omega_0, \tau_t)-) \le \delta < 1$ therefore,

$$\left\| \Phi(V^s(t; \omega_0, \tau_t)) - \Phi(V'^s(t; \omega_0, \tau_t)) \right\| \le \delta \left\| V^s(t + \epsilon; \omega_0, \tau_t) - V'^s(t + \epsilon; \omega_0, \tau_t) \right\|. \qquad (29)$$

Therefore for any $\epsilon > 0$, by the contraction mapping theorem it follows that: a) $V_\epsilon^s(t; \omega_0, \tau_t) = V^s(t; \omega_0, \tau_t) = \overline{V}_0^s(\omega_1)$. b) The ask prices are stationary and independent of τ_t. The proof for the bid prices is similar. $\qquad \square$

Proof of Theorem 7: From part b) of Theorem 6, let $S_{\epsilon,t}(\omega_0, \tau_t) = \widehat{S}(\omega_0)$ for all ω_0, the stationary ask prices. Therefore $F_{\epsilon,t}^a(x) = F^a(x)$, $a = b, s$ are stationary distributions and $V^a(t; \omega_0, \tau_t) = \overline{V}^a(\omega_0)$, $a = b, s$. Then:

$$\overline{V}^s(\omega_0) = \int_{\widehat{S}(\omega_0)}^{\infty} [u(\omega_0 + \frac{\widehat{S}(\omega_0) + x}{2} - qY) + Eu(\omega_1 + Y)]dF^b(x)$$

$$+ F^b(\widehat{S}(\omega_0)) \left[q_{t+\epsilon}u(\omega_0 - qY) + (1 - \tau_{t+\epsilon})\overline{V}^s(\omega_0) \right]$$

$$= \int_{\widehat{S}(\omega_0)}^{\infty} [u(\omega_0 + \frac{\widehat{S}(\omega_0) + x}{2} - qY) + Eu(\omega_1 + Y)]dF^b(x)$$

$$+ F^b(\widehat{S}(\omega_0))\overline{V}^s(\omega_0).$$

Therefore:

$$\int_{\widehat{S}(\omega_0)}^{\infty} [u(\omega_0 + \frac{\widehat{S}(\omega_0) + x}{2} - qY) + Eu(\omega_1 + Y) - \overline{V}^s(\omega_0)]dF^b(x) = 0,$$

implying that:

$$[u(\omega_0 + \frac{\widehat{S}_0(\omega_0) + x}{2} - qY) + Eu(\omega_1 + Y) - \overline{V}^s(\omega_0)]dF^b(x) \text{ for almost all } x \in (\widehat{S}(\omega_0), \infty).$$

In particular, for $x > \widehat{S}_0(\omega_0)$ $dF^b(x) = 0$ since u is strictly increasing hence

$$u(\omega_0 + \widehat{S}_0(\omega_0) - qY) + Eu(\omega_1 + Y) = \overline{V}^s(\omega_0).$$

Or:

$$u(\omega_0 + \widehat{S}_0(\omega_0) - qY) + Eu(\omega_1 + Y) = u(\omega_0 - qY) + Eu(\omega_1 + \rho + Y),$$

that implies that $\widehat{S}(\omega_0)$ is the price of the last trading opportunity when scheduled announcements are credible (see (15)). The same argument applies to the buyers.

As before the expected volume at t, is given by:

$$v_\tau = \Pr\{(\omega_0^b, \omega_0^s) : \widehat{B}_0(\omega_0^b) \geq \widehat{S}_0(\omega_0^s)\}$$

$$= \Pr\{(\omega_0^b, \omega_0^s) : B_0(\omega_0^b) \geq S_1(\omega_0^s)\} = v,$$

since the bid and ask prices are stationary. □

Proof of Corollary 1: By theorem (2) and (7) for some $\bar{t} \in (0,1]$, $v_\tau = v > v_t$ for all $t < \bar{t}$.

Therefore

$$\int_0^1 v_\tau dt > \int_0^1 v_t dt.$$

□