Strong Cosmic Censorship in de Sitter spacetimes

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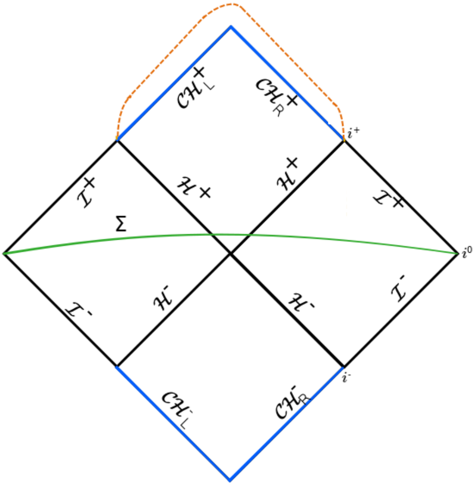
Outline

Example: Reissner-Nordstrom



Cauchy horizon

Cauchy problem: initial value problem with initial data on spacelike hypersurface Σ.

The Cauchy horizon is the boundary of the region where the solution to the Cauchy problem is unique.

Infinitely many ways to smoothly extend over the Cauchy horizon as a solution of the equations of motion - which one to choose?

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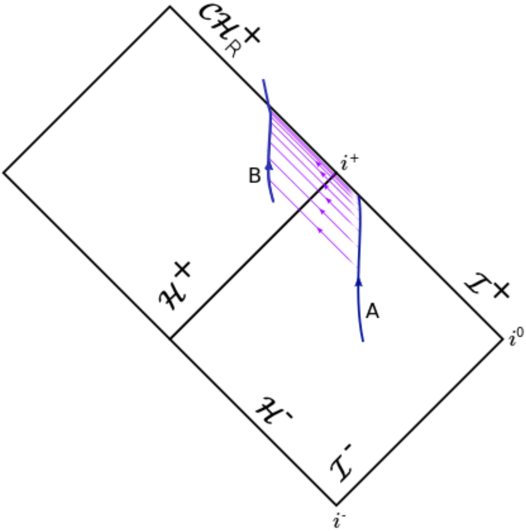
If it is unstable then it needs finely tuned initial data to form, and is unlikely to be physical.

Blue-shift effect



What is Strong Cosmic Censorship (SCC)?

Conjecture

(Penrose) In some class of suitable initial data for the Einstein equations the maximal development is, generically, inextendible.

Motivation: GR is a classical and deterministic theory but predictability breaks down if it is possible to extend the maximal development in multiple different ways. SCC restores predictability without having to resort to poorly understood physics.

Linear problem

Look at the behaviour of a massless scalar field or linearized gravitational perturbations (or gravito-electromagnetic for Einstein-Maxwell theory).

Here, concentrate on the proxy problem of massless scalar wave equation on fixed background

$$\Box_g \psi = 0. \tag{1}$$

What do we mean by 'inextendible'?

C^0 formulation: inextendible with C^0 metric.

- Linear results: for charged black holes, can extend ψ or the metric continuously across $\mathcal{CH}^+$. [Mcnamara (1978), Dafermos (2005), Franzen (2016)]

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C^2 formulation

Inextendible with C^2 metric.

- Poisson and Israel (1990): "null dust" model. The Hawking mass diverges due to backreaction, which implies that $R_{abcd}R^{abcd}$ also diverges. Cauchy horizon becomes a curvature singularity.

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Ori (1991): an observer can experience finite total tidal distortion even when metric is not in C^2!

Weak solutions

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- Depends on the minimal regularity, can make sense of solutions less regular than C^2 as weak solutions.
- E.g. shocks in compressible fluids.
- For a quasilinear second order pde, multiply by a smooth, compactly supported test function. Integrate by parts to eliminate the second order derivatives. A *weak solution* satisfies this equation for any arbitrary test function.

$$0 = \int d^4x \, f^{ab} R_{ab} \sim \int d^4x (-\partial f \Gamma + f \Gamma\Gamma).$$

Christodoulou's formulation

For GR, a weak solution has locally square integrable Christoffel symbols.

Christodoulou's version of the conjecture: generically, the maximal development is inextendible as a spacetime with locally square integrable Christoffel symbols. [Christodoulou (2009)]

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For the scalar field: $\psi \notin H^1_{loc}$ at $\mathcal{CH}^+$.

(The Sobolev space H^1_{loc} consists of square integrable functions for which the gradient is also locally square integrable.)

i.e. Energy of ψ diverges at the Cauchy horizon.

Linear version of SCC for $\Lambda = 0$

Linear version of Christodoulou's formulation respected for Reissner-Nordstrom and Kerr [Luk & Oh '17, Dafermos & Shlapentokh-Rothman '17]: ψ uniformly bounded but derivatives transversal to $\mathcal{CH}^+$ blow-up.

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Important things to note:

- Behaviour of ψ at $\mathcal{CH}^+$ depends on the behaviour at the event horizon
- Decay at event horizon is inverse *polynomial*, determined by power-law tails.
- Blue shift effect introduces an *exponential* in time factor.

What happens to $\mathcal{CH}^+$ (non-linear problem)?

- In Einstein-Maxwell theory with a massless scalar field, the metric extends continuously (in spherical symmetry) but is not in C^2 [Poisson & Isreal '90, Dafermos '05, Luk & Oh (2017)]
- In Kerr $\mathcal{CH}^+$ is still a null boundary and g extends beyond it continuously [Dafermos & Luk '17].

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It is expected that Christodoulou's formulation does hold.

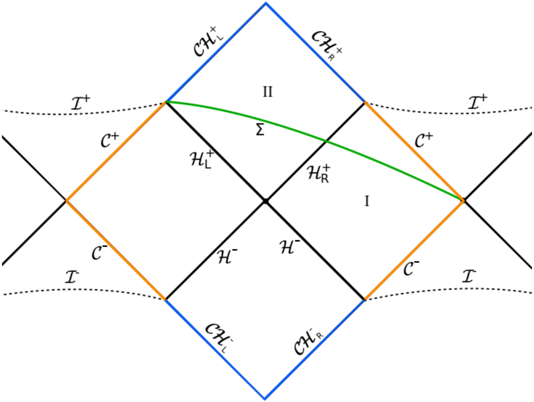
Cosmological constant

$\Lambda < 0$: perturbations outside an AdS black hole decay logarithmically (much slower than for $\Lambda = 0$) [Holzegal & Smulevici (2013)]. This probably makes the instability at the Cauchy horizon worse, so Christodoulou's version of SCC is expected to hold. But C^0 version still false [Kehle (2018)]

Assume $\Lambda > 0$ from now on.

Black hole-de Sitter spacetimes

Kerr-de Sitter:



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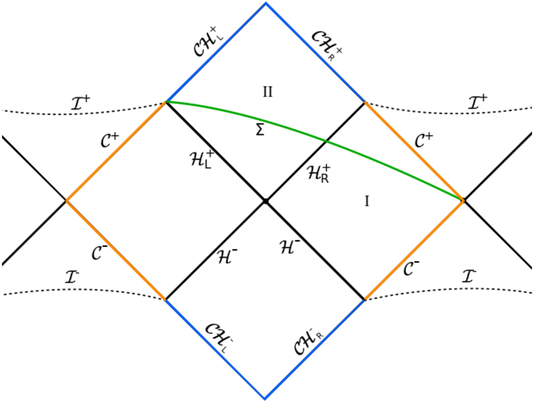
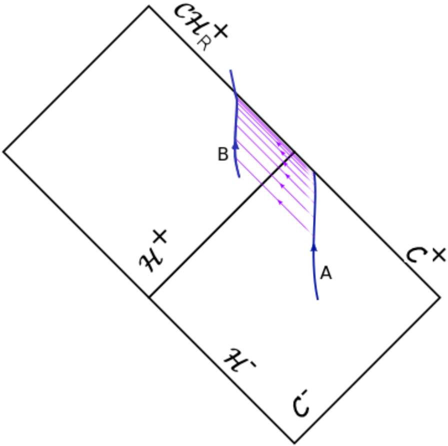


Perturbations decay exponentially in the exterior. Red-shift effect due to the Cosmological horizon (not present for $\Lambda = 0$) that competes with the blue-shift effect.


$\mathcal{CH}_R^+$
B
A
$\mathcal{H}^+$
$\mathcal{C}^+$
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What are QNMs?

- Solutions ψ with time dependence $e^{-i\omega t}$, with ω complex and $Im(\omega) < 0$.
- 'Ingoing' at the event horizon and 'outgoing' at the cosmological horizon (smooth at both horizons).
- Mathematically, take the Fourier transform, then the quasinormal frequencies ω are the poles of the Green's function.

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QNM determine the late-time behaviour of ψ in the exterior when the initial data is smooth. This determines the behaviour at the Cauchy horizon. [Hintz & Vasy '17]

Behaviour near the Cauchy horizon

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Slowest decay comes from this quasinormal frequency: generic perturbations decay as $e^{-\alpha t}$.
Introduce double null coords (U, V). Cauchy horizon is at $V = 0$, interior of black hole is $V < 0$.

Near the Cauchy horizon, generic linear perturbations are proportional to $(-V)^{\beta}$, where

$$\beta = \frac{\alpha}{\kappa_-}, \qquad (2)$$

κ_- is the surface gravity of the Cauchy horizon.

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Brady, Moss and Myers (1998): overlooked outgoing radiation.

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Similar extendibility for the metric [Hintz & Vasy (2016), Costa, Girão, Natário, Silva (2017), Dafermos & Luk (2017)]

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Non-linear results: numerical confirmation that the scalar field results are true with backreaction in spherical symmetry [Luna *et al* (2018)]

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To show SCC is respected only need to find one quasinormal mode that has

$$-Im(\omega) < \frac{\kappa_-}{2} \quad \Rightarrow \quad \beta < \frac{1}{2}. \tag{3}$$

Then a *generic* perturbation of the initial data does not belong to H^1_{loc}.

We studied QNM of Kerr-de Sitter with large angular frequency, *i.e.* proportional to $e^{im\phi}$ with $m \gg 1$.

Photon sphere modes

'Trapped' geodesics lead to slower decay rates, e.g. unstable trapping in Schwarzschild, Kerr etc. and stable trapping in Kerr-Ads, microstate geometries, ultra-compact neutron stars...

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Hamilton-Jacobi eq for null geodesics:

$$g^{\mu\nu}\partial_\mu S \partial_\nu S = 0 \tag{4}$$

then

$$\frac{\partial S}{\partial x^\mu} \equiv p_\mu \quad \text{and} \quad p^\mu = \frac{dx^\mu}{d\tau} \tag{5}$$

Geometric optics approximation

Use ansatz

$$S = -Et + j\phi + R(r) + \Theta(\theta). \tag{6}$$

The geodesic equation in the equatorial plane reduces to

$$\dot{r}^2 = V(r) \tag{7}$$

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Geometric optics approximation: for large $\ell = m$:

$$\omega = m\Omega_c - i(n + \frac{1}{2})\lambda \tag{8}$$

$\Omega_c (= 1/b = E/j)$ is the orbital angular velocity of the orbit and λ is the Lyapunov exponent [Cardoso *et al.* '09, Yang *et al.* '12 ...] More accurate as $\ell \to \infty$.

Results in Kerr-de Sitter

For any non-extremal black hole, the slowly decaying QNM place an upper bound on β, with the result

$$\beta < \frac{1}{2}$$

Indications that Christodoulou's formulation of SCC holds in Kerr-de Sitter!

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Supported by numerical evidence, found photon sphere quasinormal modes numerically the massless scalar field and linearized gravitational perturbations.
For the linearized gravitational perturbations, looked at rate of blow up of a component of the Weyl tensor that is gauge invariant. Fast enough to suggest SCC is respected, and this conclusion cannot be altered by trying to use a different gauge.

Rough SCC in RNdS

How to rescue SCC in Einstein-Maxwell theory with $\Lambda > 0$?

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Dafermos & Shlapentokh-Rothman (2018): consider rough initial data. If one only requires that the initial data is in H^1_{loc}, i.e. has finite local energy on the initial hypersurface Σ, then the solution to $\Box\psi = 0$ generically has infinite energy on a hypersurface intersecting the Cauchy horizon transversally. This suggests Christodoulou's version is respected.

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More generally, determine whether the smoothness of the solution (in the sense of Sobolev spaces) generically gets worse at the Cauchy horizon.

Brady, Moss & Myers' argument only holds for non-smooth initial data, not even C^1 at the event horizon. [Dias, Reall, Santos (2018)]

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- There is a qualitative difference between Einstein-Maxwell system and vacuum Einstein equations: SCC seems to be violated in the first but respected in the second!
- Seem to be able to recover SCC by allowing non-smooth initial data.

Future work

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- Quantum corrections: calculate the renormalized stress-energy tensor.

Thank you!