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Thermodynamics of relativistic multifluids

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Multifluids in compact objects

The fundamental fields of the theory are the four-currents. The equation of state is assumed to be given in terms of a master function

$$\Lambda = \Lambda(n^2_{xy}) \qquad n^2_{xy} := -n^\nu_x n_{y\nu}$$

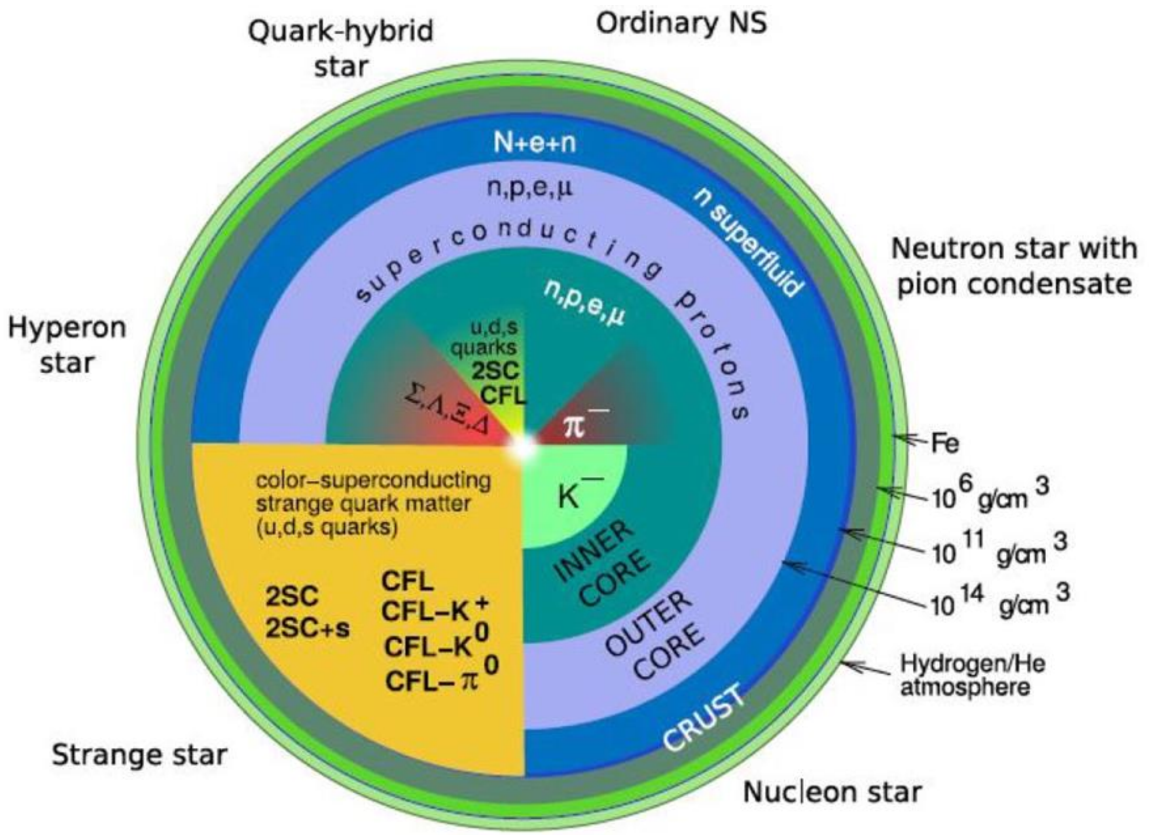
One is naturally lead to define the momenta

$$\mu^x_\nu := \frac{\partial \Lambda}{\partial n^\nu_x}$$

The energy-momentum tensor is then given by

$$T^\nu{}_\rho = \Psi \delta^\nu{}_\rho + \sum_x n^\nu_x \mu^x_\rho$$

$$\Psi = \Lambda - \sum_x n^\rho_x \mu^x_\rho$$



The importance of thermodynamics

Equation of state

The Lagrangian density (phenomenological scalar) Λ vs Thermodynamic potentials (ensemble averages) $\mathcal{U}, \mathcal{F}, \ldots$

Connection with other formalisms

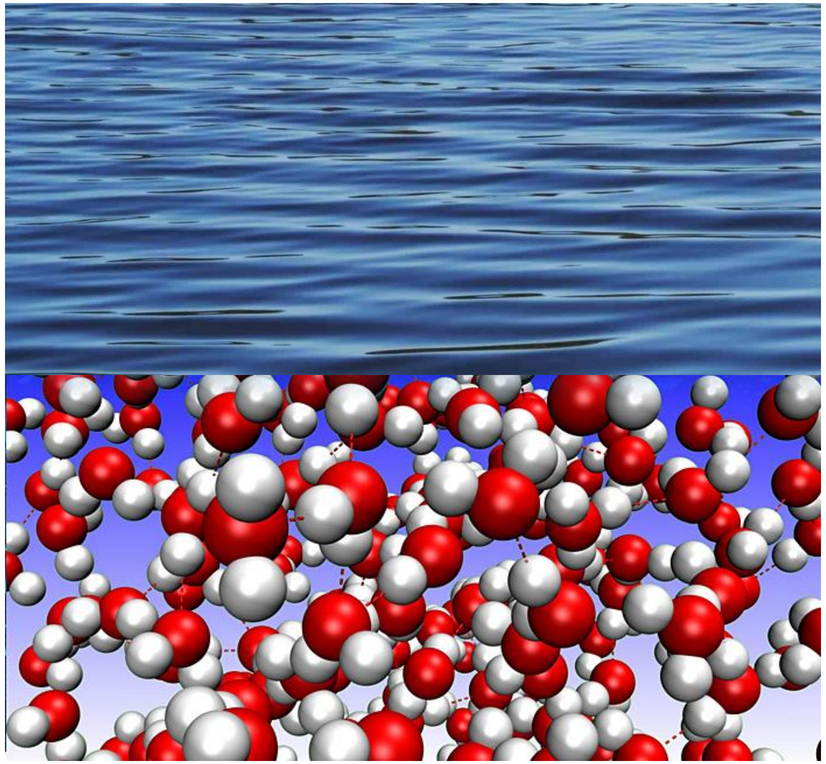
Currents of particles and entropy n_x^ν, s^ν vs Ordinary thermodynamic variables (parameters of the density matrix) Θ, μ_x, w_ν

Statistical interpretations of hydrodynamic phenomena

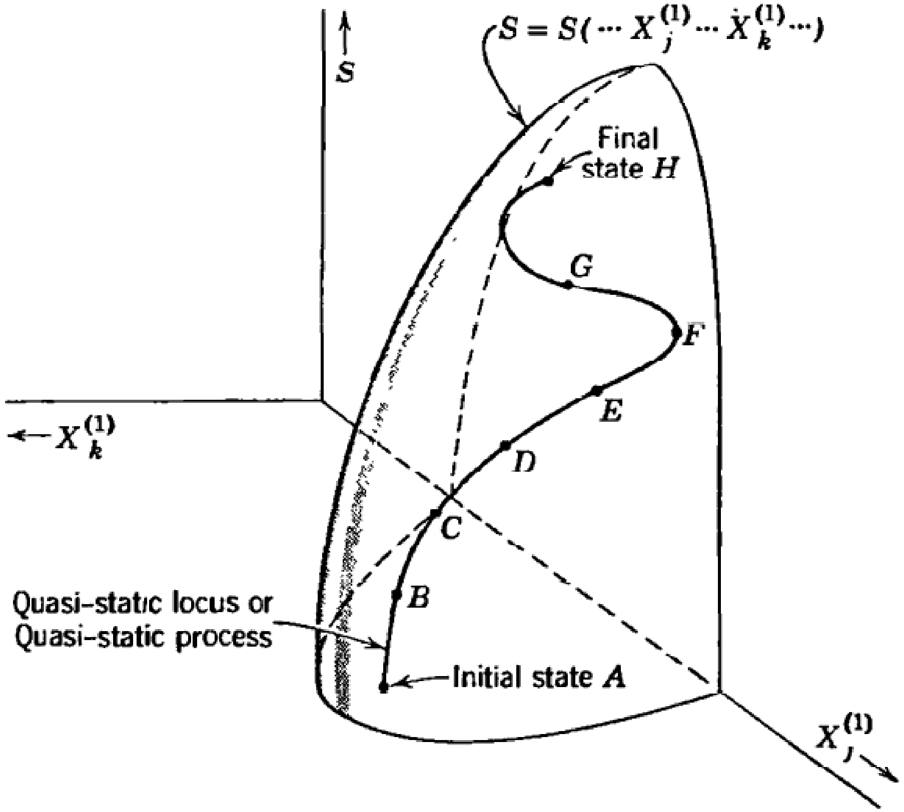
- Vanishing dissipation → Maximum entropy
- Friction → Entropic force
- Long life of superfluid currents → Non-ergodic behaviour

Equilibrium thermodynamics

Only few macroscopic independent degrees of freedom are necessary to describe some large (nearly infinite) systems



These degrees of freedom are the macroscopic manifestation of constants of motion (fundamental or effective)

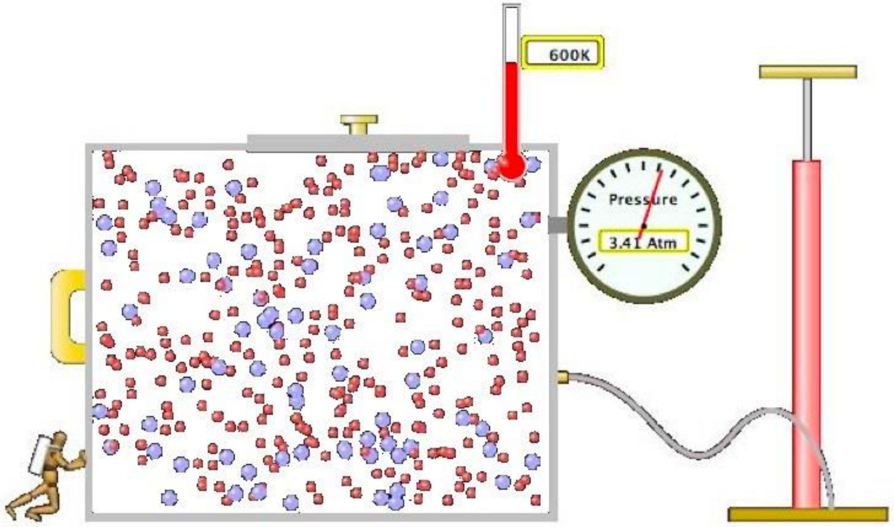


Temperature, pressure, chemical potentials and the entropy itself are uniquely defined only in equilibrium!

Thermodynamics of normal fluids

A system of particles, with different species, confined in a box with adiabatic walls.

Constants of motion (without transfusion):
- Energy
- Position of the walls
- Number of particles of each species



$$S = S(E, V, N_x)$$

If there is transfusion $\sum_x \alpha_x X_x \rightleftharpoons \sum_x \beta_x X_x$

There is a constant of motion less for each reaction $\frac{dS}{d\xi} = 0 \quad \Longrightarrow \quad \sum_x \alpha_x \mu_x = \sum_x \beta_x \mu_x$

Non-equilibrium processes

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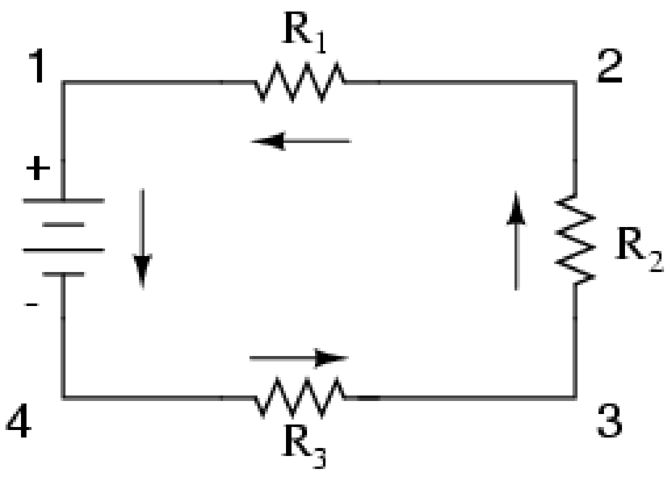
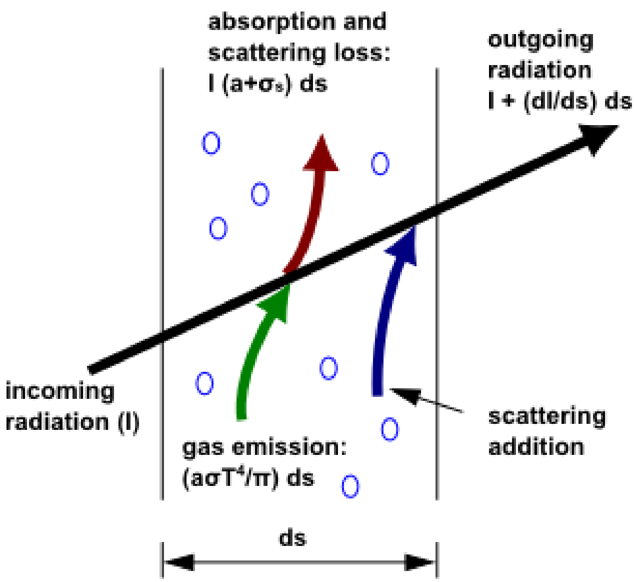
3) Full many-body theory: all the microscopic degrees of freedom
- Molecular dynamics
- Liouville equation

Multifluids as non-equilibrium processes

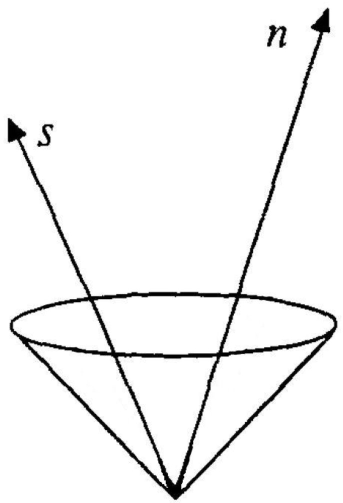
Electric current in a circuit



Radiative transport



Models for heat conduction



- In these cases the relative currents do not exist in thermodynamic equilibrium
- Near equilibrium assumption is required to avoid the need for full kinetic theory
- Relative currents are associated with entropy production, they are an irreversible process
- Quantities like pressure and temperature may not be uniquely defined
- A proper notion of chemical equilibrium between relatively flowing species may not exist

Superfluids are different

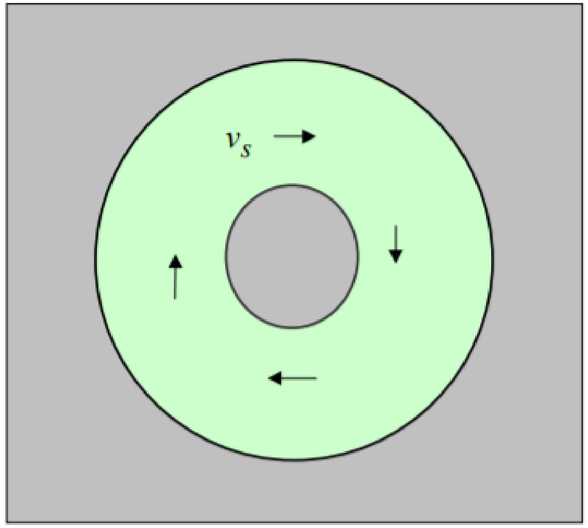
There is an order parameter $\psi = |\psi| e^{i\phi}$

The gradient of its phase is traditionally interpreted as a "velocity" (in the zero-temperature limit it is)

$$\mathbf{v}_S = \frac{\hbar}{m} \nabla \phi \qquad \Longrightarrow \qquad \nabla \times \mathbf{v}_S = 0$$

Its circulation is quantized and can survive for thousands of years

$$\int_\gamma m \mathbf{v}_S \cdot d\mathbf{l} = zh \qquad z \in \mathbb{Z}$$

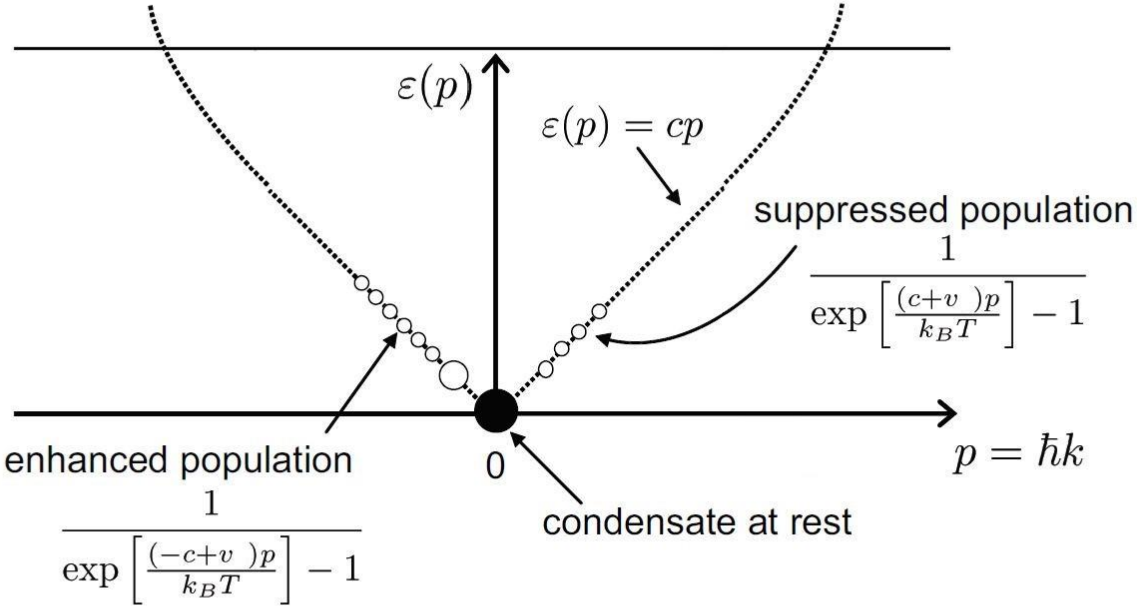


There is a new constant of motion, therefore relative currents can exist also in thermodynamic equilibrium!

Landau's approach to bosonic superfluidity

We choose the frame in which the order-parameter wave function is the state with zero momentum (the gradient of its phase is zero).

$$\mathbf{J}_0 = \int \mathbf{p}\, n(\mathbf{p})\, d_3 p \qquad E_0 = \int \varepsilon(\mathbf{p})\, n(\mathbf{p})\, d_3 p$$



The excitations thermalize in a given frame, which is not necessarily the one of the condensate, so in general it will move with a velocity $\mathbf{v}_N$

$$n(\mathbf{p}) = \frac{1}{h^3} \frac{1}{e^{\beta(\varepsilon(\mathbf{p}) - \mathbf{v}_N \cdot \mathbf{p})} - 1}$$

For the partition function to converge we need to require that

$$\varepsilon(\mathbf{p}) - \mathbf{v}_N \cdot \mathbf{p} > 0 \implies$$

Landau superfluidity criterion

$$v_N < \min_p \frac{\varepsilon(p)}{p}$$

Isotropy implies that the mass current is proportional to the average velocity of the excitations

$$\mathbf{J}_0 = \rho_N \mathbf{v}_N$$

Landau's equation of state

The equation of state has to include the dependence on the currents (it is computed in the superfluid frame):

$$dE_0 = \mu d\rho + Td(\rho\mathfrak{s}) + \boxed{(\mathbf{v}_N - \mathbf{v}_S) \cdot d\mathbf{J}_0}$$

New entry!

This adds a term to the Euler relation:

$$P = -E_0 + T\rho\mathfrak{s} + \mu\rho + \boxed{\rho_N(\mathbf{v}_N - \mathbf{v}_S)^2}$$

New entry!

The stress tensor in a generic frame:

Superfluid mass-current flux

$$\Pi_{jk} = \delta_{jk}P + \boxed{\rho_S v_{Sj} v_{Sk}} + \boxed{\rho_N v_{Nj} v_{Nk}}$$

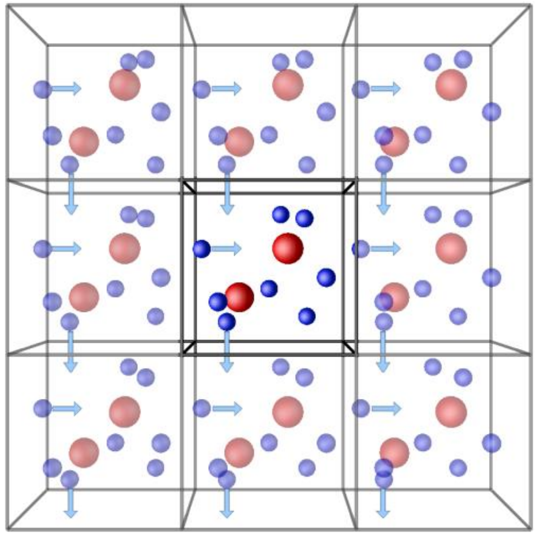
Normal mass-current flux

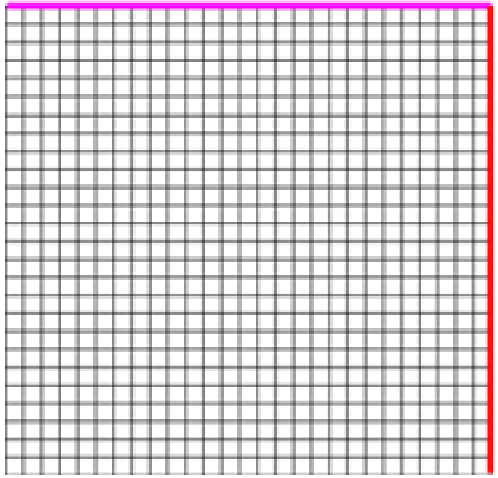
We are in the framework of equilibrium thermodynamics

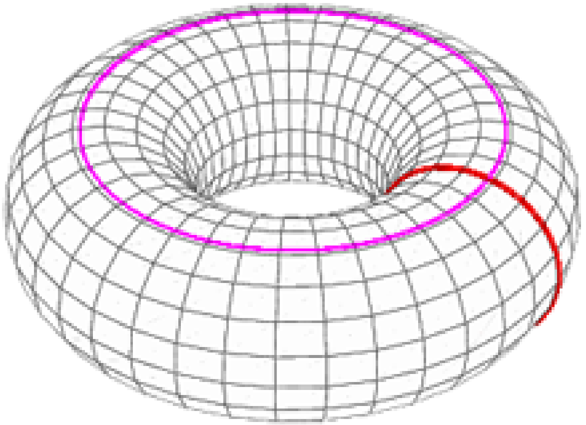
- No ambiguity in the definition of the temperature: it is the temperature of the environment
- No ambiguity in the definition of the pressure: it is minus the grand potential density
- No need for kinetic theory to study the physical properties of the fluid

Generalizing Landau's approach

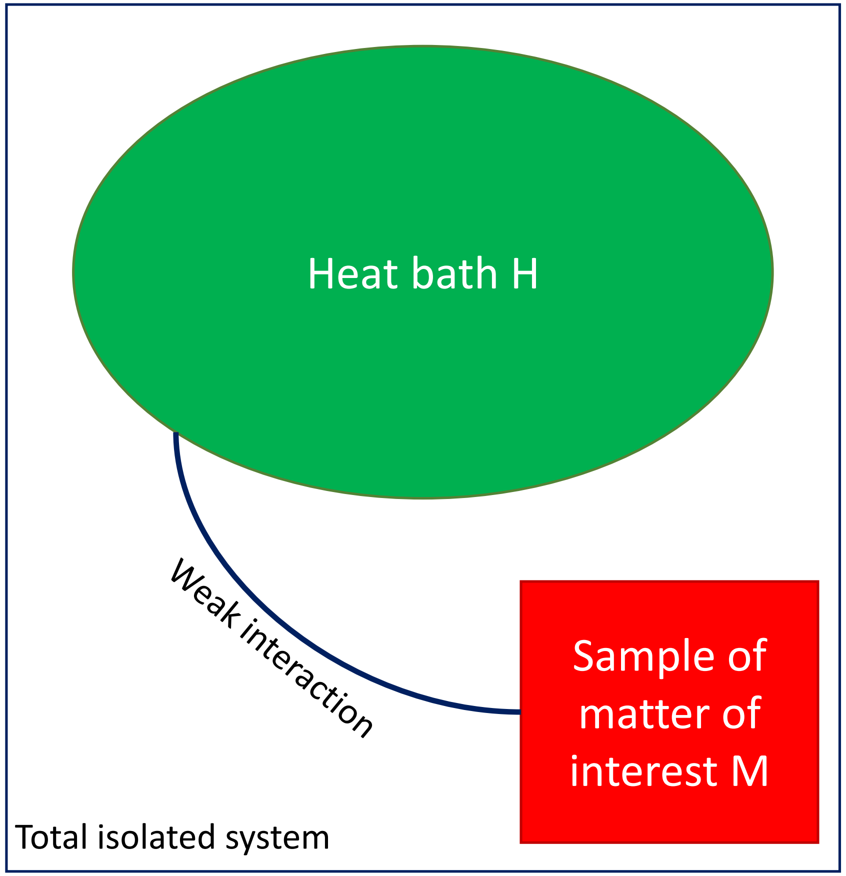
Box with periodic boundary conditions





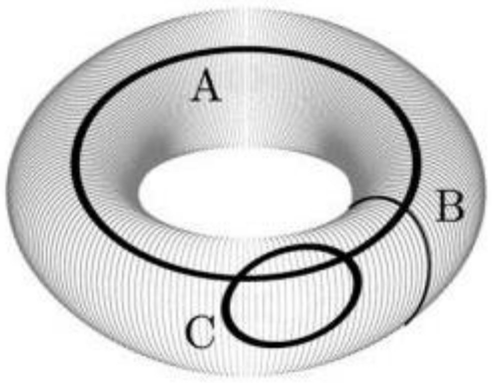


We put the box in contact with a heat bath



Essence of the multifluid thermodynamics

Constants of motion (without transfusion):
- Total H+M energy
- Volume of the box
- Number of particles of each species
- Winding numbers of the superfluid phases over non-contractable loops (like A and B, see fig.)



$$W^x_\gamma = \frac{1}{k_x} \int_\gamma \mu^x_\nu dx^\nu$$

Principle of maximum entropy:
- The entropy is at rest in the frame of the bath
- Normal species are at rest in the frame of the bath
- We discover (comparing with the canonical ensemble) what the temperature is
- We discover (enabling transfusion) what the chemical potentials are

$$\Theta = -\mu^s_\nu u^\nu_s$$

$$\mu_x = -\mu^x_\nu u^\nu_s$$

Multifluid equation of state:

$$d\mathcal{U} = \Theta ds + \sum_{A=1}^{l} \mu_A^{(T)} dn_A + \sum_{i=1}^{k} \mu_i^{(T)} dn_i^0 + \boxed{\sum_{i=1}^{k} n_i^j d\mu_j^i}$$

New entry!

Immediate applications

1) We can make the Legendre transformation of the internal energy density

Helmholtz free energy

$$\mathcal{F} = \mathcal{U} - \Theta s$$

Grand potential

$$\mathcal{K} = \mathcal{U} - \sum_x n_x^0 \mu^{x0}$$

New entry potential

$$\mathcal{E} = \mathcal{U} - \sum_{i=1}^{k} n_i^j \mu_j^i$$

$$\mathcal{E} = -\Lambda \qquad \Longrightarrow$$

We have a direct thermodynamic interpretation for the master function

2) The connection with other formalisms becomes transparent

$$d\mathcal{U} = \Theta ds + \sum_{A=1}^{l} \mu_A^{(T)} dn_A + \sum_{i=1}^{k} \mu_i^{(T)} dn_i^{(T)} + \sum_{i,h=1}^{k} \frac{Y_{ih}}{2} d(w_i^\nu w_{h\nu})$$

3) Allowing for transfusion, we recover usual concepts from chemistry

Affinity: $\mathbb{A} := -\dfrac{d\mathcal{F}}{dn^0} = \sum_{x \neq s} (\alpha_x - \beta_x) \mu_x^{(T)}$

Le Châtelier's principle:

$$\mathbb{A} > 0 \quad \Longrightarrow \quad n_{eq}^0 > n^0 \quad \Longrightarrow \quad \frac{dn^0}{dt} > 0$$

$$\mathbb{A} < 0 \quad \Longrightarrow \quad n_{eq}^0 < n^0 \quad \Longrightarrow \quad \frac{dn^0}{dt} < 0$$

Conclusions

The thermodynamic interpretation of the hydrodynamic quantities cannot be ambiguous and represents the contact with microphysics

It can be used to make bridges between different hydrodynamic descriptions and to verify if these are the same theory written in terms of different variables

It represents a useful tool for the consistent implementation of dissipation

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Thank you for your attention!

Appendices

Formalisms

Many different formalisms are used in neutron star physics to describe the multicomponent hydrodynamics of the interior. The most important ones are

- Carter-Prix formulation (Lagrangian-type, relativists' favorite)
- Kobyakov-Pethick formulation (Hamiltonian-type, nuclear physicists' favorite)
- Son-Gusakov formulation (Landau-type, matter structurists' favorite)

Going from one to the other is not always simple.

Each formulation **seems** to capture only a part of the problem and **seems** to be unfit for the others.